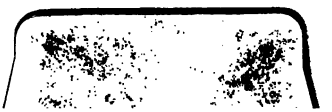

A

NEW SYSTEM

OF

ARITHMETIC:

Containing

MANY HUNDRED EXAMPLES,

Constructed upon a Plan

Entirely Original.

EXPRESSLY DESIGNED FOR THE ASSISTANCE OF TEACHERS;

And calculated to facilitate the

PROGRESS OF THE PUPIL.

BY J. WALKER.

A New Edition,

WITH AN APPENDIX

By W. RUSSELL,

Author of the Companion to every Treatise on Arithmetic.

LONDON:

PRINTED FOR JOHN SOUTER, SCHOOL LIBRARY,
73, ST. PAUL'S CHURCH YARD,
By J. and C. Adlard, 23, Bartholomew-Close.

1823.

PREFACE.

"ARITHMETIC," says Dr. Watts, "is the more valuable as it is the more exact, easy, and short; and the art lies in giving as few rules as possible, and clearly explaining them." Convinced of the justness of the Doctor's remark, the author has not only condensed the *general system* of Arithmetic, but, in the very condensation, has, he trusts, rendered it, by means of Elucidations, Notes, and Remarks, clearly intelligible to the most limited capacity.

As soon as a pupil has mastered his Numeration Table, he exults in the acquisition gained to the stock of knowledge he had previously acquired; and, when he aims at the *proof* of his *first operation* in Arithmetic, no conqueror's breast ever swelled with more pride than does that of the young arithmetician.

As he advances from the separation of one quantity from another by *Subtraction*, to the expeditious process of collecting quantities by *Multiplication*, and separating them by *Division*, he rejoices in new and more complicated discoveries; but his joy is the joy of one who has arrived at *truth* founded on *demonstration*, through every step of which the *evidence* has been *intuitive* and *infallible*.

In the application, too, of the four first Rules to *Reduction, Arithmetical Fractions, Decimal Fractions, Practice, Interest*, &c. the young arithmetician has the same certain species of demonstrative evidence; and thus, while the faculties of *attention*,

memory, *association*, and *abstraction*, have been exercised, the *reasoning powers*, also, have been called forth, and received a proper direction in the pursuit of *scientific knowledge*. A study in which youth proceed with so much certainty cannot, therefore, fail to be both amusing and instructive.

Proportion, in every system that has ever come under the author's notice is either divided into the *Single* and *Double Rule-of-Three*, or into *Direct*, *Inverse*, and *Compound Proportion*; but here it is given under *one general rule*, and from inspection it will be found that this rule clearly teaches the method of calculating *any kind* of Proportion, and thereby rendering *nugatory* the distinctions in *common use*.

The Rules of each branch of the science are, it is presumed, laid down with scientific perspicuity; the questions arranged with such attention to their conditions, as well as such familiar illustrations of their solutions, as cannot fail to elucidate, in a very popular manner, the respective processes in the different operations, that serve as *models* for the exemplification of the rules.

W. RUSSELL.

5, *Northumberland Street, Mary-le-bone; February*, 1823.

ARITHMETICAL TABLES.

NUMERATION.

1	Units.
21	Tens.
321	Hundreds.
4321	Thousands.
54321	Tens of Thousands.
654321	Hundreds of Thousands.
7654321	Millions.
87654321	Tens of Millions.
987654321	Hundreds of Millions.

MONEY.

4	Farthings	1	Penny.
12	Pence	1	Shilling.
5	Shillings	1	Crown.
6	Shil. and 8*d.*	1	Noble.
13	Shil. and 4*d.*	1	Mark.
10	Shillings	1	Angel.
21	Shillings	1	Guinea.
27	Shillings	1	Moidore

FARTHINGS.

q.	*d.*
4	1
5	1¼
6	1½
7	1¾
8	2
9	2¼
10	2½
11	2¾
12	3
13	3¼
14	3½
15	3¾
16	4
17	4¼
18	4½
19	4¾
20	5
21	5¼
22	5½
23	5¾
24	6
25	6¼
26	6½

PENCE.

s.	*s.*	*d.*
12	1	0
20	1	8
24	2	0
30	2	6
36	3	0
40	3	4
48	4	0
50	4	2
60	5	0
70	5	10
72	6	0
80	6	8
84	7	0
90	7	6
96	8	0
100	8	4
108	9	0
110	9	2
120	10	0
130	10	10
132	11	0
140	11	8
144	12	0

SHILLINGS.

s.	£	*s.*
20	1	0
30	1	10
40	2	0
50	2	10
60	3	0
70	3	10
80	4	0
90	4	10
100	5	0
110	5	10
120	6	0
130	6	10
140	7	0
150	7	10
160	8	0
170	8	10
180	9	0
190	9	10
200	10	0
300	15	0
400	20	0
500	25	0
600	30	0

MULTIPLICATION.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

WEIGHTS AND MEASURES.

TROY WEIGHT.

24 Grains make 1 Pennyweight.
20 Pennyweights 1 Ounce.
12 Ounces ....1 Pound.

AVOIRDUPOISE WEIGHT.

16 Drams make 1 Ounce.
16 Ounces ...1 Pound.
28 Pounds ....1 Quarter.
4 Quarters ....1 Cwt. or 112 lb.
20 Hun.-Weight 1 Ton.
19½ Hun.-Weight 1 Fother.

APOTHECARIES' WEIGHT.

20 Grains make 1 Scruple.
3 Scruples ....1 Dram.
8 drams .....1 Ounce.
12 Ounces ....1 Pound.

CLOTH MEASURE.

2¼ Inches make 1 Nail.
4 Nails .....1 Quarter.
4 Quarters ....1 Yard.
3 Quarters ..1 Flemish Ell.
5 Quarters....1 English Ell.
6 Quarters....1 French Ell.

LONG MEASURE.

3 Barley-corns 1 Inch.
12 Inches ....1 Foot.
3 Feet .....1 Yard.
6 Feet .....1 Fathom. [Perch.
5 Yards & a half 1 Rod, Pole, or
40 Poles .....1 Furlong.
8 Furlongs....1 Mile.
3 Miles .....1 League
60 Miles .....1 Degree

WINE MEASURE.

2 Pints .....1 Quart.
4 Quarts.....1 Gallon.
10 Gallons ....1 Anchor.
18 Gallons ....1 Rundlet.
42 Gallons ....1 Tierce.
63 Gallons ....1 Hogshead.
2 Hogsheads ..1 Pipe or Butt.
2 Pipes or 4 Hhds. 1 Tun.

ALE AND BEER MEASURE.

2 Pints make ..1 Quart.
4 Quarts .....1 Gallon.
9 Gallons .....1 Firkin of Ale or Beer.
2 Firkins .....1 Kilderkin.
4 Firk. or 2 Kild. 1 Barrel.
1 ½ Bar. or 54 Gal. 1 Hhd. of Beer.
2 Barrels ....1 Puncheon.
3 Barrels ....1 Butt.

DRY MEASURE.

2 Pints make..1 Quart.
2 Quarts.....1 Pottle.
4 Quarts.....1 Gallon.
2 Gallons ....1 Peck.
4 Pecks .....1 Bushel.
2 Bushels ....1 Strike.
4 Bushels ....1 Coom.
2 Cooms or 8 Bu. 1 Quarter.
36 Bushels ....1 Chaldron.
5 Quarters ....1 Wey or Load.
2 Weys .....1 Last of Corn.

COAL MEASURE.

4 Pecks make .1 Bushel.
3 Bushels ....1 Sack.
12 Sacks or 36 Bu. 1 Chaldron.
21 Chaldrons ..1 Score.

TIME.

60 Seconds ... 1 Minute.
60 Minutes ....1 Hour.
24 Hours .....1 Day.
7 Days .....1 Week.
4 Weeks.....1 Month.
13 Months, 1 Day, 6 hours; or 365 Days, 6 hours, } 1 Year.

Thirty days hath September,
April, June, and November;
February hath *Twenty-eight* alone;
All the rest have *Thirty-one*,
Except leap-year, and then's the time
February's days are *Twenty-nine*.

SQUARE MEASURE.

144 Inches ....1 Square Foot.
9 Square Feet 1 Square Yard.
100 Feet .....1 Sq. of Flooring.
272¼ Feet.....1 Rod.
40 Rods or Per. 1 Rood.
4 Roods.....1 Acre.
640 Acres .....1 Square Mile.
30 Acres .....1 Yard of Land.
100 Acres .....1 Hide of Land.

SOLID MEASURE.

1728 Cubic Inches 1 Solid Foot.
27 Cubic Feet .1 Cubic Yard.

HAY.

A Load contains 36 Trusses.
A Truss weighs 56 Pounds.

ARITHMETIC.

ARITHMETIC is the art of computing by numbers; the rules upon which all its operations depend being Numeration, Addition, Subtraction, Multiplication, and Division.

NUMERATION.

NUMERATION teaches to express numbers by words or figures, or to read and write any sum or number.

Write in figures the following numbers.

Thirty-nine.
One hundred and seventy-three.
Six hundred and twenty-seven.
Nine hundred and sixteen.
Two thousand, five hundred, and forty-four.

Seventy-six thousand, nine hundred, and sixty-nine.

Ninety-six thousand, five hundred, and eighty.

One hundred and eighty-nine thousand, seven hundred, and ninety-four.

Eight hundred and seventy-six thousand, four hundred, and fifty-eight.

One million, three hundred and thirty-three thousand, eight hundred, and forty-five.

Six million and a half.

Ninety-six millions, seven hundred and sixty thousand, five hundred, and thirty-eight.

Five hundred millions, seven thousand, and eighty.

Seven hundred and seven millions, eight thousand, nine hundred, and six.

Nine hundred millions and one thousand.

One hundred millions, one hundred thousand, and one hundred.

Write in words the following numbers.

86	940509
470	2206308
6056	7600027
20709	91020312
84625	101010101
110101	

SIMPLE ADDITION.

SIMPLE ADDITION teaches to collect several numbers of the same denomination into one sum.

RULE.

Place the numbers under each other so that units may stand under units, tens under tens, &c. and draw a line under them.

Add up the figures in the row of units, and find how many tens are contained in their sum.

Set down what remains above the tens, or, if nothing remains, a cypher, and carry as many ones to the next row as there were tens.

Add up the second together with the number carried in the same manner as the first; and proceed thus till the whole is finished.

Method of Proof.

Draw a line below the uppermost number and suppose it cut off.

Add all the rest together, and set their sum under the number to be proved.

Add this last-found number and the uppermost line together; and, if the sum be the same as that found by the first addition, the work is right.

236	346	248	268
322	234	323	234
416	843	442	442
294	429	286	235
364	308	474	319
461	432	142	424
143	754	333	346
174	432	372	437
2410			

248	364	345	246
244	236	335	327
428	847	454	242
349	622	343	373
426	834	668	462
218	116	546	143
335	345	654	453
364	279	657	873

3470	3468	3673
4323	3434	4346
7985	8356	6758
4864	4647	4649
6642	7494	3937
1755	3775	6746
4431	2294	3564
6737	7254	8754

3642	3647	4872
4234	3436	3435
7622	4378	8694
4934	8787	7463
6786	6493	8276
2808	2642	7484
3616	4264	4648
4368	5839	9654

37286	47564	46795
24356	43544	45345
65248	54397	26837
72937	97163	73726
20629	49686	32944
41404	38094	65606
20636	65328	47443
12453	17536	24657
42337	34252	83442
73459	86972	94582

57342	48892	57982
45469	34547	65458
53688	72726	89337
60326	40692	46694
74874	27465	67866
86968	84082	28978
65629	41277	39607
74837	60476	97736
38259	38735	64324
63975	97684	68785

48763	47947	48754
86454	65348	54536
70976	27944	87284
44687	59737	58653
53749	65826	46594
64938	48414	37461
40704	37566	38673
63967	26620	52434
24525	68545	24365
94375	63526	86945

47694	57984	58745
64548	46545	46549
79674	69737	68476
56896	75986	75964
67354	63674	63793
38066	84963	84826
15474	41697	60754
49563	74862	56986
28425	42536	42652
67354	64375	62473

4675453	5384253
5454837	4654436
3547416	8497644
8479656	7575753
5846734	6484891
4937538	8368015
1625642	6296333
5562035	5468564
4546142	2654364
3546142	6792542

5643273	5374235
4545745	4545633
9754583	3367894
4683849	7434752
6496733	6796936
8254647	9683434
3578137	7242241
6264522	5473436
6426784	5455674
3634726	6485742

246757	193464
246545	235348
9737	944
865986	44737
3674	3826
34963	414
1697	86566
4862	620
832536	627545
665678	904326

178645	137486
236549	234548
74476	65674
63964	4896
4793	254
826	166
754	6474
617986	34563
652	658425
349734	954302

4687943	5679462
6464548	3546545
5979674	8069737
4756896	5375986
2567854	7463674
4888066	4084963
6915474	8141697
4849563	3774862
3628425	9542536
6354865	8432736

5768425	4697583
6546549	5436454
5468476	7670976
6975964	2944687
4863793	3653749
8684826	4764938
4960754	9540704
7656986	2363967
4842652	3624525
3169754	2985674

5947685	5786573
3465348	6454536
9227944	4987284
4059737	7658663
8165826	5746594
7348414	9837461
9537566	6528673
4726620	2952434
3468545	5834365
7434794	6407239

6938795	6754893
3546549	4565458
9268476	9789337
8075964	7446694
7463793	8667866
9084826	9328978
6560754	6239607
8756986	7597736
7242652	6364324
2796573	2187302

Add 1875, 1213, 6192, 2595, and 6432, together.

Add 1687, 2113, 4906, 2981, and 4271, together.

Add 1374, 1228, 6197, 2575, and 6393, together.

Add 3752, 2238, 8026, 9647, 6453, 3636, and 7465, together.

Add 3842, 2337, 3286, 6144, 8758, 9475, and 4634, together.

What will 2684, 2239, 5677, 8348, 3736, and 3415, make, added together?

Add two thousand, three hundred, and seventy-five;—two thousand, one hundred, and twenty-eight;—six thousand, eight hundred, and six;—two thousand, seven hundred, and twenty-four;—eight thousand, three hundred, and forty-two;—and six thousand, four hundred, and thirty-two, together.

What will 1645, 1238, 49, 8676, 37, and 9326, make, added together?

Add one thousand, four hundred, and thirty-five;—two thousand, two hundred, and thirty-seven;—seventy-nine;—six thousand, nine hundred, and forty-six;—seven hundred and thirty-eight;—and six thousand, four hundred, and twenty-six, together.

Add 47654, 34347, 86986, 77479, 90508, 64834, 45846, and 31485, together.

A man had six purses, containing as follows: In one, 1432 guineas; in another, 2328; in another, 494; in another, 3683; in another, 3495; and in another, 2641: How many guineas had he in the whole?

Add 38476, 45348, 67974, 46846, 34917, 67968, 36947, and 94398, together.

Suppose in one town there were 1842 inhabitants, in another 2237, in another 1846, in another 3378, in another 2539, and in another 6432: What number would they make, all added together?

What will 1426, 2328, 3494, 1286, 2892, and 2314, make, added together?

Suppose a merchant shipped to London 1432 bushels of wheat; to Hull, 1238; to Bristol, 2047; to Liverpool, 3486; to Boston, 3229; and to Lynn, 3741: What was the number of bushels in all?

Add 85274362, 67457668, 94672986, 49503897, 64824524, 86601616, 74736858, 82533987, 56347675, 61623102, 86955424, 74742263, and 74578784, together.

SIMPLE SUBTRACTION.

SIMPLE SUBTRACTION teaches to find the difference between any two numbers of the same denomination by taking the less from the greater.

RULE.

Place the less number under the greater, so that units may stand under units, tens under tens, &c. and draw a line under them.

Begin at the right hand, and take each figure in the lower line from the figure above it, and set down the remainder.

But if the figure in the lower line be greater than that above it, add ten to the upper one, and then take the lower figure from it. Set down the remainder, and carry one to the next lower figure; with which proceed as before; and so on till the whole is finished.

Method of Proof.

Add the remainder to the lowest number, and if the sum be equal to the greatest, the work is right.

From 8997789546
Take 4117400532

Rem. 4880389014

Proof 8997789546

From 9987999975
Take 3032301042

From 8650448775
Take 2051964882

From 1245984291
Take 298896768

From 846247680
Take 276372639

From 11252711898
Take 2763869005

From 14339638971
Take 7561869588

From 12864658599
Take 6903819576

From 1884244212
Take 990369198

From 1115526294
Take 123456789

From 4985398449	From 1759783788
Take 1085469696	Take 992698083
From 1381385898	From 696638997
Take 695346993	Take 108943965
From 1278412191	From 10919991519
Take 691327386	Take 3241596546

In these sums the lower line is to be subtracted from the upper and its remainder twice.

21309105	16948407
8523642	7263603
18390930	14606580
7356372	5842632
30585155	61536830
52234062	24614732

40512647	105721847
17362563	45309363
114533573	118379844
52060715	52618264
98116955	168806617
39246782	72345693
128017179	106560617
56896524	45672693
90645980	240771830
36258392	96308732

The lower line to be subtracted from the upper and its remainder three times.

1387924517	281104993
396549862	86493844

1264784053	211067760
389164324	65958675
2458921210	2438601452
787676363	696743272
3070786368	114940462
944857344	32840182
1850214808	1273550610
569296864	382065183
2762195667	2122487068
789198762	653072944

The lower line to be subtracted from the upper and its remainder four times.

2881464444	454122462
640325432	106852344

1728557559	1154413155
384123902	274860275
1345187363	2083822164
310427853	463071592
1163091553	1967461389
268405743	437213642

The lower line to be subtracted from the upper and its remainder five times.

1267573241	901020336
230467862	168941313
2046308781	2272401136
372056142	426075213
1350836256	715728871
257302144	130132522

3239048176	540857961
607321533	103020564

The lower line to be subtracted from the upper and its remainder six times.

2163986779	1304398693
341682123	200676722
4243608029	792561025
670043373	126809764
4526332661	1920268649
730053655	303200313
3287950275	3972926049
526072044	627304113

A person was born in the year 1593, and died in the year 1620, how old was he?

A bridge was built in the year 1665, how old will it be if it should stand till the year 4662?

What is the difference between the numbers 14768496 and 104074281?

Two churches were built, one in the year 1296, and the other in the year 1359: how many years was one built before the other?

What number added to 2885418 will make 8854173?

How much does 71278200 exceed 21519486?

What number added to 58346496 will make 152155161?

If a person born in the year 1782 should live till the year 1881, how old would he be?

SIMPLE MULTIPLICATION.

SIMPLE MULTIPLICATION is a compendious method of Addition, which teaches to find the amount of any given number of one denomination repeated a certain number of times?

The number to be multiplied is called the multiplicand. The number you multiply by is called the multiplier. The number found after the work is finished is called the product. Both the multiplier and multiplicand are in general called terms or factors.

RULE.

When the multiplier does not exceed 12, multiply each figure in the multiplicand by it, beginning at the unit's place, reckoning every time how many tens there are in the product; Set down the remainder, if any, and carry as many ones as there were tens to the product of the next figure.

464524407	388032966
2	2
929048814	
31895838	22899141
3	3

218939508 4	2446883 4
236921508 4	175009761 5
137215701 5	145848969 6
14894631 6	11591631 6
13668489 7	112283289 7
8697096 8	9935946 8

846336879 8	981372879 8
8731053 9	963215487 9
632315448 9	3895696953 10
9833896863 10	97408396763 10
787978953 11	534399543 11
631896093 11	32473836 12

6622163901	57246336
12	12

When the multiplier exceeds 12, and any two numbers being multiplied together will produce it, the given number may be multiplied by one of those numbers, and the product by the other.

4236556032

$4 \times 4 = 16$

16940224128

4

67784896512

Multiply 3865527137 by 18.

279850833 by 21.

3647832741 by 24.

3179502639 by 25.

2844049959 by 27

25013709945 by 35.

1382488293 by 36.

Multiply 2041832736 by 42.
1525730355 by 45.
1449571644 by 48.
1210524102 by 56.
79039431 by 72.
345409423506 by 144.

When the multiplier consists of several figures, multiply by each of them separately, beginning with the right-hand figure, and place the first figure of every product under the figure multiplied by.

Note.—When there are cyphers at the right-hand of the multiplier or multiplicand, or both, they may be omitted till the work is done, and afterwards added to the product.

7341677	6530527
15	24
36708385	
7341677	
110125155	5321649
	35
6120833	7921563
25	36

3950273	1057316
45	48
4673947	4103297
66	36
5793024	5120756
77	88
3045687	45364900
99	1500
5752340	27351960
360	480
34296960	61738270
240	450

72923800	81726900
8800	25000
12629930	15625
350	16499
312500	156250
13299	364350
7812500	1953125
1128450	451236
390625	48828125
225648	1204836
12207031250	244140625
18192450	3409648

305175781250	39062500
232768250	12561500
152587890625	2441406250
16553625	14096350
1220703125	6103515625
2819215	11638425
30517578125	3814697265625
13276836	126214436
2384185791015625	19073486328125
1419430415	152428835

476837158203125
1209715248

What would 31250 amount to, taken 13245 times?

How many letters would there be in 1512350 books, supposing each book to contain 19531250 letters?

What number divided by 195312500 will make 35122500?

Suppose in one bushel of wheat there were 488281250 grains, how many would there be in 12048450 bushels?

SIMPLE DIVISION.

SIMPLE DIVISION is a compendious method of subtraction, which teaches to find how often one number is contained in another of the same denomination.

The number to be divided is called the dividend. The number you divide by is called the divisor. The number of times the dividend contains the divisor is called the quotient.

If the dividend contains the divisor any number of times and some part or parts over, those parts are called the remainder.

RULE.

When the divisor does not exceed 12, find how often it is contained in the first figure, or in the first two or three figures of the dividend, as may be necessary, and set the quotient underneath. Carry the overplus, if any, to the next figure in the dividend, as so many tens, and see how often the divisor is contained therein: set down the quotient, and proceed as before, till the whole is finished.

2) 77813190

38906595

2) 99537264

3) 2031281417

3) 2301278744

3) 20591966531

4) 3502312164

4) 3968350020

4) 3536267328

5) 482829529

5) 3835199164

5) 4469690119

6) 401195592

6) 584692884

6) 508423910

7) 549974130

7) 688161921

7) 467654361

7) 587994567

8) 542271240

8) 819078512

8) 613631880

8) 2399432184

9) 626012153

9) 6098931296

9) 707109596

9) 431895653

10) 488065950

10) 695686050

10) 7943980140

11) 4288526164

11) 1062225064

11) 9824031865

12) 8232801660

12) 813115476

12) 694423368

12) 1148109984

When the divisor exceeds 12, and is the product of two numbers, the dividend may be divided by one of those numbers, and the quotient by the other.

14 { 2) 6833086848
   { 7) 3416543424
     488077632

Divide 59785425 by 15.
1387101024 by 16.
628023375 by 21.
2274404184 by 24.
1562890464 by 27.
1921067820 by 28.
16429578759 by 33.
2023268625 by 35.
24375053628 by 36.
32594088744 by 42.
424530855 by 49.
458421930 by 54.
5970309849 by 63.
6488241210 by 66.
6106223880 by 72.
3431614824 by 88.
938225486 by 99.
8478809460 by 108.
1190456685 by 121.
10244426940 by 132.
97495160112 by 144.

LONG DIVISION.

When the divisor consists of several figures, find how often it is contained in as many figures of the dividend as are just necessary. Multiply the divisor by the quotient figure, and place the product under the figures of the dividend above mentioned. Subtract this product from that part of the dividend under which it stands, and being down the next figure in the dividend. Divide this number so increased as before; and so on till the whole is finished.

```
26 ) 2585 (99
     234
     ---
      245
      234
      ---
       11
       --

31 ) 2000 (
```

```
47 ) 4528 (

53 ) 3360 (

65 ) 6026 (

79 ) 12276 (

86 ) 14469 (

97 ) 44308 (
```

236) 149987 (

384) 262827 (

425) 409596 (

589) 509658 (

672) 98805 (

793) 131996 (

809) 3621830 (

934) 2787802 (

1365) 467001 (

2389) 13532712 (

3278) 7985614 (

4613) 35487042 (

5264) 19077642 (

6437) 9835927 (

7192) 183373084 (

8375) 82461891 (

9825) 273324766 (

29345) 1084691556 (

18846) 59082837 (

43916) 2981732052 (

50763) 173762890 (

68107) 1154825118 (

79328) 5377186098 (

81309) 5443166432 (

96153) 6479190598 (

264379) 99516548400 (

451982) 397032055974 (

When cyphers are annexed to the divisor, cut them off, and the same number of figures from the right-hand of the dividend.

If any thing remains after the division, place the figures cut off to the right-hand of it, and it will be the true remainder.

696400) 2536290621 (

65740000) 501924901527 (

71846000) 6731467309231 (

643250000) 43305519772441 (

How many half dozens are contained in 287919792?

Find the quotient and remainder of 860750153, divided by nine.

What is the eighth part of 311914512?

How many dozens are there in 5974500168?

What number is that which multiplied by 12, will make 8243904276

How many times is 8473 contained in 27894762?

Find the quotient and remainder of 8565414378 divided by 86725.

COMPOUND ADDITION.

COMPOUND ADDITION teaches to collect several numbers of different denominations into one sum.

RULE.

Place the numbers so that those of the same denomination may stand directly under each other, and draw a line below them.

Add up the figures in the lowest denomination, and find how many units or ones of the next higher denomination are contained in their sum.

Write down the remainder, and carry the ones to the next denomination, which add up in the same manner as before.

Proceed thus through all the denominations to the highest, whose sum, together with the several remainders, will give the answer required.

The method of proof is the same as in Simple Addition.

£	s.	d.
1	10	$4\frac{1}{4}$
3	6	$4\frac{1}{4}$
5	12	$6\frac{1}{2}$
1	1	$1\frac{1}{4}$
3	4	$6\frac{1}{2}$
14	14	$10\frac{3}{4}$

£	s.	d.
1	17	$6\frac{1}{2}$
2	5	$3\frac{1}{4}$
6	13	$7\frac{1}{2}$
1	1	$1\frac{1}{4}$
6	14	$3\frac{3}{4}$

£	s.	d.
1	4	$7\frac{1}{4}$
2	10	$3\frac{3}{4}$
6	8	$5\frac{1}{2}$
1	1	$2\frac{3}{4}$
6	11	$8\frac{3}{4}$

£	s.	d.
1	14	$7\frac{1}{2}$
2	19	$0\frac{3}{4}$
4	9	$8\frac{1}{2}$
2	11	$2\frac{3}{4}$
8	13	$4\frac{1}{2}$

£	s.	d.
1	11	$5\frac{1}{2}$
0	3	$4\frac{1}{4}$
8	15	$6\frac{1}{4}$
1	1	$1\frac{1}{2}$
6	6	$9\frac{1}{4}$

£	s.	d.
1	14	$3\frac{1}{2}$
0	19	$10\frac{3}{4}$
6	7	$10\frac{1}{2}$
2	12	$2\frac{3}{4}$
7	12	$4\frac{1}{4}$

£	s.	d.
346	12	$4\frac{1}{4}$
325	4	$2\frac{3}{4}$
167	1	10
648	17	$6\frac{1}{2}$
787	6	$8\frac{1}{4}$
39	17	$5\frac{3}{4}$
578	7	$11\frac{1}{2}$
453	4	$3\frac{1}{4}$
622	1	$1\frac{1}{4}$

£	s.	d.
477	16	$8\frac{3}{4}$
686	4	4
395	4	$2\frac{1}{4}$
787	1	4
44	6	$8\frac{1}{4}$
698	11	$7\frac{1}{4}$
846	8	$7\frac{1}{2}$
542	3	$2\frac{3}{4}$
123	1	$1\frac{1}{4}$

£	s.	d.
368	14	$6\frac{3}{4}$
573	5	$8\frac{1}{4}$
44	6	$8\frac{1}{4}$
948	12	4
64	8	$10\frac{3}{4}$
897	4	8
28	17	$6\frac{1}{4}$
443	4	$2\frac{1}{2}$
221	1	$1\frac{1}{4}$

£	s.	d.
386	17	$10\frac{1}{2}$
661	5	5
57	8	6
694	7	$10\frac{3}{4}$
68	6	$9\frac{1}{2}$
876	14	$8\frac{1}{4}$
98	12	6
543	4	$2\frac{1}{2}$
763	14	$6\frac{3}{4}$

£	s.	d.
479	14	$4\frac{3}{4}$
863	17	$3\frac{1}{2}$
61	18	2
684	4	$0\frac{1}{2}$
971	17	$1\frac{3}{4}$
85	8	$4\frac{3}{4}$
798	12	8
534	2	$3\frac{1}{2}$
498	16	$6\frac{3}{4}$

£	s.	d.
375	16	$11\frac{1}{4}$
248	11	$4\frac{1}{4}$
364	7	10
958	18	$8\frac{1}{2}$
9	4	6
67	17	$5\frac{3}{4}$
896	15	$11\frac{1}{4}$
454	4	$2\frac{1}{4}$
603	3	$0\frac{1}{4}$

£	s.	d.
468	18	$7\frac{1}{2}$
789	10	6
773	7	$7\frac{1}{4}$
98	12	11
984	2	$4\frac{1}{2}$
72	4	10
738	17	$6\frac{3}{4}$
543	4	$2\frac{1}{4}$
987	15	$8\frac{3}{4}$

£	s.	d.
371	16	$2\frac{1}{4}$
498	11	$3\frac{1}{2}$
894	3	4
9	4	$9\frac{3}{4}$
478	16	$7\frac{1}{2}$
17	1	10
648	17	$9\frac{1}{2}$
453	4	$3\frac{3}{4}$
897	18	$9\frac{1}{2}$

£	s.	d.
37	12	$2\frac{1}{2}$
79	6	5
6	7	$7\frac{2}{4}$
75	4	$10\frac{3}{4}$
2	16	$4\frac{1}{4}$
84	7	$10\frac{1}{2}$
8	12	$6\frac{3}{4}$
43	4	$3\frac{1}{4}$
18	3	$1\frac{1}{4}$

£	s.	d.
37	16	$4\frac{1}{4}$
88	11	0
24	10	$2\frac{3}{4}$
2	2	$0\frac{1}{2}$
38	5	$1\frac{1}{4}$
96	2	$7\frac{3}{4}$
8	6	$8\frac{1}{2}$
42	2	$3\frac{1}{4}$
21	2	$1\frac{1}{4}$

£	s.	d.
37	16	9¼
27	4	6½
2	4	5¼
44	16	6¾
98	8	9¼
4	14	5
79	6	11¾
43	4	3½
28	1	2¼

£	s.	d.
10	18	6½
17	13	4½
17	2	9¾
6	17	7½
15	14	8¼
18	19	4¾
7	17	8½
15	14	4¾
18	14	8¾

£	s.	d.
40	8	6¼
37	13	2¼
65	12	9½
87	16	6
8	19	10¼
76	5	7¾
67	17	8½
55	14	3¾
36	17	4¼

£	s.	d.
40	9	6¼
34	13	8¼
49	10	10¾
66	8	4
48	16	7¼
76	17	6½
67	18	7¾
55	14	3½
42	15	4¼

£	s.	d.
50	18	$10\frac{1}{2}$
24	12	$8\frac{3}{4}$
66	13	2
98	8	$6\frac{1}{2}$
89	19	8
67	15	$7\frac{1}{2}$
96	17	6
55	13	$2\frac{1}{4}$
67	14	$8\frac{3}{4}$

£	s.	d.
50	16	$7\frac{1}{2}$
34	14	$2\frac{1}{2}$
69	16	$4\frac{1}{2}$
97	19	$9\frac{3}{4}$
64	16	$7\frac{1}{4}$
96	18	$8\frac{3}{4}$
78	19	$10\frac{1}{2}$
56	14	$4\frac{3}{4}$
94	14	$10\frac{1}{4}$

£	s.	d.
5784	13	$2\frac{1}{4}$
2109	15	$2\frac{3}{4}$
382	4	7
2109	15	$2\frac{3}{4}$
3248	2	$4\frac{3}{4}$
4337	5	$1\frac{1}{4}$
58	10	10
4337	5	$1\frac{1}{4}$
844	17	$8\frac{1}{2}$
6432	3	$6\frac{1}{2}$
9782	6	$2\frac{3}{4}$
63	8	$3\frac{3}{4}$
842	6	$4\frac{1}{2}$
4343	6	$5\frac{1}{2}$
6765	6	$5\frac{1}{4}$
4343	6	$5\frac{1}{2}$
8579	14	$1\frac{1}{2}$

£	s.	d.
6387	16	8¼
2789	16	9¾
8922	8	4¾
908	12	5¾
2789	16	9¾
4837	10	4¾
996	11	11¼
3449	7	5¼
8759	8	10¾
842	17	4¾
3449	7	5¼
8782	8	8
3037	6	11¾
4893	14	3¾
646	17	8¾
4893	14	3¾
9163	4	6¼

£	s.	d.
6387	16	8¼
8922	8	5¾
4893	14	3¾
908	12	4¾
4837	10	11¾
4893	14	3¾
4391	14	5½
996	11	10¼
8759	8	7¾
4391	14	5½
5578	13	4½
842	17	8¾
3449	7	4¼
3037	6	10¾
646	17	4¾
3449	7	4¼
4968	14	7½

What is the sum of 12*l.* 2*s.* 0¼*d.*, 18*l.* 18*s.* 10¼*d.*, 81*l.* 1*s.* 1¼*d.*, and 13*l.* 13*s.* 0½*d.*, added together?

How much will 2*l.* 7*s.* 11¼*d.*, 8*l.* 8*s.* 10½*d.*, 10*l.* 10*s.* 0*d.*, 1*l.* 1*s.* 1½*d.*, and 6*l.* 7*s.* 10¼*d.*, make, added together?

Find the amount of 20*l.* 9*s.* 6*d.*, 52*l.* 4*s.* 6*d.*,

47*l.* 15*s.* 6*d.*, 77*l.* 3*s.* 3*d.*, 22*l.* 16*s.* 9*d.*, and 22*l.* 18*s.* 7*d.*

A. owes B. for tea, 2*l.* 2*s.* 4*d.*; for rice, 6*l.* 5*s.* 9*d.*; for coffee, 5*l.* 12*s.* 3*d.*; for butter, 5*l.* 9*s.* 6*d.*; for cheese, 2*l.* 12*s.* 6*d.*; and for sugar, 6*l.* 8*s.* 7*d.*: what is the amount of the whole?

Add 2*l.* 13*s.* 6*d.*, 4*l.* 2*s.* 7*d.*, 4*l.* 16*s.* 6*d.*, 8*l.* 8*s.* 7*d.*, 2*l.* 12*s.* 4*d.*, and 4*l.* 7*s.* 4*d.*, together.

A. owes B. for sarcenet, 3*l.* 13*s.* 7*d.*; for plain silk, 8*l.* 17*s.* 6*d*; for flowered ditto, 8*l.* 18*s.* 6*d.*; for satin, 2*l.* 5*s.* 2*d.*; for kid gloves, 6*l.* 16*s.* 6*d.*; for ribbons, 3*l.* 2*s.* 4*d.*; and for lace, 6*l.* 7*s.* 5*d.*: what does he owe in all?

How much will 2*l.* 13*s.* 7¼*d.*, 1*l.* 12*s.* 2¼*d.*, 4*l.* 10*s.* 6*d.*, 2*l.* 8*s.* 8¼*d.*, 7*l.* 16*s.* 6*d.*, 3*l.* 12*s.* 1¼*d.* and 6*l.* 9*s.* 2¼*d.*, make, added together?

B. owes D. for cheese, 4*l.* 7*s.* 4¼*d.*; for butter, 6*l.* 10*s.* 0*d.*; for soap, 6*l.* 14*s.* 3¼*d.*; for coals, 7*l.* 16*s.* 8*d.*; for sugar, 7*l.* 8*s.* 4*d.*; for candles, 4*l.* 13*s.* 1¼*d*; for flour, 6*l.* 17*s.* 7*d.*, and for bread, 6*l.* 13*s.* 2¼*d.*: what does the whole come to?

What is the amount of 3*l.* 2*s.* 4*d.*, 2*l.* 15*s.* 2*d*, 8*l.* 18*s.* 6*d.*, 8*l.* 17*s.* 6*d.*, 3*l.* 13*s.* 6*d.*, 5*l.* 15*s.* 4*d.*, and 6*l.* 16*s.* 6*d.*?

Add 2*l.* 15*s.* 0*d.*, 2*l* 17*s* 6*d.*, 6*l.* 12*s.* 3*d.*,

2*l.* 13*s.* 0*d.*, 2*l.* 17*s.* 6*d.*, 4*l.* 19*s.* 9*d.*, and 2*l.* 2*s.* 6*d.*, together.

A gentleman owed to different tradesmen the following sums: 30*l.* 13*s.* 2¼*d.*, 33*l.* 14*s.* 6¼*d.*, 26*l.* 14*s.* 8¼*d.*, 33*l.* 16*s.* 9¼*d.*, 16*l.* 17*s.* 6¼*d.*, 25*l.* 14*s.* 7¼*d.*, 34*l.* 16*s.* 5¼*d.*, 16*l.* 17*s.* 6¼*d.*, 27*l.* 13*s.* 7¼*d.*, 14*l.* 17*s.* 8¼*d.*, 68*l.* 16*s.* 5¼*d.*, and 40*l.* 16*s.* 7¼*d.*: what was the amount of the whole debts?

How much money will discharge the following debts: 30*l.* 16*s.* 4¼*d.*, 24*l.* 14*s.* 3¼*d.*, 45*l.* 14*s.* 2¼*d.*, 34*l.* 6*s.* 6*d.*, 16*l.* 17*s.* 4¼*d.*, 29*l.* 19*s.* 6¼*d.*, 13*l.* 3*s.* 3¼*d.*, 24*l.* 16*s.* 4*d.*, 26*l.* 7*s.* 5¼*d.*, 17*l.* 16*s.* 7¼*d.*, 66*l.* 4*s.* 4¼*d.*, and 45*l.* 16*s.* 8¼*d.*

COMPOUND SUBTRACTION.

Compound Subtraction teaches to find the difference between any two numbers of different denominations.

RULE.

Place the less number under the greater, so

that those parts which are of the same denomination may stand directly under each other; and draw a line below them.

Begin at the right-hand, and take each number in the lower line from that above it, and set down the remainder under it.

If any number in the lower line be greater than that above it, increase the upper number by as many as make one of the next higher denomination; then subtract the lower number from the upper one, and set down the remainder. Carry the unit borrowed to the next number in the lower line, which subtract from the number above it; and proceed in like manner till the whole is finished; and the several remainders taken together will be the whole difference required.

The method of proof is the same as in Simple Subtraction.

l.	*s.*	*d.*
39	12	$10\frac{1}{4}$
24	7	$3\frac{1}{4}$
15	5	7
39	12	$10\frac{1}{4}$

l.	*s.*	*d.*
50	19	6
32	16	8

l.	*s.*	*d.*
25	6	$1\frac{3}{4}$
12	17	$9\frac{3}{4}$

l.	*s.*	*d.*
28	0	4
18	9	$2\frac{1}{4}$

l.	*s.*	*d.*
36	14	$8\frac{3}{4}$
23	7	$3\frac{3}{4}$

l.	*s.*	*d.*
27	18	$1\frac{3}{4}$
16	8	$10\frac{1}{2}$

l.	*s.*	*d.*
35	2	$5\frac{1}{4}$
18	17	$9\frac{1}{4}$

l.	*s.*	*d.*
51	5	2
36	18	8

l.	*s.*	*d.*
12	18	$0\frac{1}{4}$
3	6	$10\frac{1}{2}$

l.	*s.*	*d.*
43	0	$7\frac{1}{2}$
23	18	$8\frac{1}{2}$

l.	*s.*	*d.*
29	6	0
18	15	$9\frac{1}{2}$

l.	*s.*	*d.*
29	5	$6\frac{1}{2}$
17	16	$3\frac{1}{4}$

In these sums the lower line is to be subtracted twice from the upper and its remainder.

l.	*s.*	*d.*
8	2	$3\frac{3}{4}$
3	9	$6\frac{3}{4}$
4	12	9
3	9	$6\frac{3}{4}$
1	3	$2\frac{1}{4}$

l.	*s.*	*d.*
16	15	$11\frac{1}{4}$
6	14	$4\frac{1}{2}$

l.	*s.*	*d.*
6	10	$0\frac{3}{4}$
2	3	$4\frac{1}{4}$

l.	*s.*	*d.*
21	11	$9\frac{1}{4}$
8	12	$8\frac{1}{2}$

l.	*s.*	*d.*
8	0	$6\frac{1}{4}$
3	4	$2\frac{1}{2}$

l.	*s.*	*d.*
7	15	$10\frac{3}{4}$
3	6	$9\frac{3}{4}$

l.	*s.*	*d.*
23	16	$6\frac{3}{4}$
9	10	$7\frac{1}{2}$

l.	*s.*	*d.*
11	8	$0\frac{1}{4}$
4	11	$2\frac{1}{2}$

l.	*s.*	*d.*
65	8	$5\frac{1}{4}$
26	8	$4\frac{1}{2}$

l.	*s.*	*d.*
18	10	$10\frac{3}{4}$
7	1	$9\frac{2}{4}$

l.	*s.*	*d.*
43	1	$4\frac{1}{4}$
17	4	$6\frac{1}{2}$

l.	*s.*	*d.*
9	17	$3\frac{3}{4}$
4	4	$6\frac{3}{4}$

l.	*s.*	*d.*
2	16	$11\frac{3}{4}$
1	2	$9\frac{1}{2}$

l.	*s.*	*d.*
44	5	$0\frac{3}{4}$
18	19	$3\frac{3}{4}$

l.	*s.*	*d.*
156	12	$0\frac{3}{4}$
67	2	$3\frac{3}{4}$

l.	*s.*	*d.*
174	5	$6\frac{3}{4}$
74	13	$9\frac{3}{4}$

l.	*s.*	*d.*
891	18	$10\frac{1}{4}$
356	15	$6\frac{1}{2}$

l.	*s.*	*d.*
133	7	$2\frac{1}{4}$
53	6	$10\frac{1}{2}$

l.	*s.*	*d.*
116	8	$10\frac{1}{4}$
46	11	$6\frac{1}{2}$

l.	*s.*	*d.*
192	5	$11\frac{1}{4}$
76	18	$4\frac{1}{2}$

l.	*s.*	*d.*
99	6	$4\frac{1}{4}$
39	14	$6\frac{1}{2}$

l.	*s.*	*d.*
2318	16	$10\frac{3}{4}$
993	15	$9\frac{3}{4}$

The lower line to be subtracted three times.

l.	*s.*	*d.*
73	11	$10\frac{1}{2}$
22	1	$6\frac{3}{4}$

l.	*s.*	*d.*
22	9	$7\frac{1}{4}$
6	8	$5\frac{1}{2}$

l.	*s.*	*d.*
575	8	$8\frac{3}{4}$
164	8	$2\frac{1}{2}$

l.	*s.*	*d.*
217	15	$0\frac{1}{4}$
62	4	$3\frac{1}{2}$

l.	*s.*	*d.*
4	7	$4\frac{1}{4}$
1	4	$11\frac{1}{2}$

l.	*s.*	*d.*
42	12	$8\frac{1}{2}$
12	15	$9\frac{3}{4}$

l.	*s.*	*d.*
11	13	$6\frac{1}{2}$
3	10	$0\frac{3}{4}$

l.	*s.*	*d.*
89	6	$10\frac{3}{4}$
25	10	$6\frac{1}{2}$

l.	*s.*	*d.*
35	4	9
8	16	$2\frac{1}{4}$

l.	*s.*	*d.*
22	14	$4\frac{1}{2}$
6	16	$3\frac{3}{4}$

l.	*s.*	*d.*
34	6	$8\frac{3}{4}$
9	16	$2\frac{1}{2}$

l.	*s.*	*d.*
22	1	$0\frac{1}{2}$
6	12	$3\frac{3}{4}$

l.	*s.*	*d.*
25444	2	$8\frac{1}{2}$
7633	4	$9\frac{3}{4}$

l.	*s.*	*d.*
21576	1	$10\frac{1}{2}$
6472	16	$6\frac{3}{4}$

l.	*s.*	*d.*
20577	19	$0\frac{1}{4}$
5879	8	$3\frac{1}{2}$

l.	*s.*	*d.*
6877	8	$8\frac{1}{4}$
1964	19	$7\frac{1}{2}$

l.	*s.*	*d.*
33094	13	$7\frac{1}{4}$
9455	12	$5\frac{1}{2}$

l.	*s.*	*d.*
12165	17	$9\frac{3}{4}$
3475	19	$4\frac{1}{2}$

The lower line to be subtracted four times.

l.	*s.*	*d.*
17	13	$5\frac{1}{4}$
4	1	$6\frac{3}{4}$

l.	*s.*	*d.*
2784	13	$8\frac{1}{4}$
618	16	$4\frac{1}{2}$

l.	*s.*	*d.*
64	4	$8\frac{1}{4}$
12	16	$11\frac{1}{4}$

l.	*s.*	*d.*
15	1	$5\frac{1}{4}$
3	9	$6\frac{3}{4}$

l.	*s.*	*d.*
15555	16	$8\frac{1}{4}$
3456	17	$0\frac{1}{2}$

l.	*s.*	*d.*
10692	13	$6\frac{1}{4}$
2467	10	$9\frac{3}{4}$

l.	*s.*	*d.*
53994	6	$5\frac{1}{2}$
12460	4	$6\frac{3}{4}$

l.	*s.*	*d.*
31218	18	$11\frac{1}{4}$
6987	10	$10\frac{1}{2}$

The lower line to be subtracted five times.

l.	*s.*	*d.*
517	3	0
96	19	$3\frac{3}{4}$

l.	*s.*	*d.*
20	19	$1\frac{3}{4}$
3	16	$2\frac{1}{2}$

l.	*s.*	*d.*
164	9	8
30	16	$9\frac{3}{4}$

l.	*s.*	*d.*
38299	9	$4\frac{1}{4}$
6963	10	$9\frac{1}{2}$

l.	*s.*	*d.*
8933	3	0
1674	19	$3\frac{3}{4}$

l.	*s.*	*d.*
9276	14	$5\frac{3}{4}$
1686	13	$6\frac{1}{2}$

l.	*s.*	*d.*
22912	9	$3\frac{1}{4}$
4165	16	$11\frac{1}{2}$

l.	*s.*	*d.*
34467	9	8
6462	13	$0\frac{3}{4}$

What is the difference between 2*l.* 8*s.* $10\frac{1}{4}$*d.* and 12*l.* 0*s.* $0\frac{1}{4}$*d.*?

What change is due out of a 20*l.* note, after buying goods which cost 5*l.* 13*s.* 6*d.*?

Borrowed 33*l.* 10*s.* $2\frac{1}{4}$*d.*, and have paid, in part, 22*l.* 19*s.* $11\frac{1}{4}$*d.*: what remains due?

What is the difference between 57*l.* 18*s.* $9\frac{1}{4}$*d.* and 38*l.* 16*s.* $10\frac{1}{4}$*d.*

A tradesman bought goods which cost 114*l.* 6*s.* $2\frac{1}{4}$*d.*, and paid, in part, 100*l.* 18*s.* $9\frac{1}{4}$*d.*: what remains due?

If, from a thousand pounds, there be sub-

tracted 988*l.* 10*s.* 8¼*d.*, what will be the remainder?

What is the difference between 28*l.* 4*s.* 3*d.* and ten guineas and a half?

Borrowed 19 guineas, and paid at different times 6*l.* 19*s.* 4¼*d.*, 1*l.* 7*s.* 2½*d.*, and 2*l.* 1*s.* 3¼*d.*: what remains due?

What sum of money added to 137*l.* 19*s.* 3¼*d.* will make 157*l.* 1*s.* 2¼*d.*?

What sum added to 9½ guineas will make 20*l.* 9*s.* 8¼*d.*?

What is the difference between 100 guineas and 93*l.* 10*s.* 8¼*d.*?

What sum added to 50 guineas will make 67*l.* 15*s.* 7*d.*?

COMPOUND MULTIPLICATION.

COMPOUND MULTIPLICATION teaches to find the amount of any given number of different denominations repeated a certain number of times.

RULE.

Place the multiplier under the lowest denomination of the multiplicand.

Multiply the number in the lowest denomination by the multiplier, and find how many integers of the next higher denomination are contained in the product, and write down what remains.

Carry the integers thus found to the product of the next higher denomination, with which proceed as before, and so on through all the denominations to the highest; and the product, together with the several remainders taken as one number, will be the whole amount required.

Proof by Division.

l.	*s.*	*d.*
7	12	$9\frac{1}{2}$
		2
15	5	7

l.	*s.*	*d.*
5	14	7
		3

l.	*s.*	*d.*
3	6	$10\frac{1}{4}$
		4

l.	*s.*	*d.*
1	18	$2\frac{3}{4}$
		5

l.	*s.*	*d.*
2	17	$3\frac{1}{2}$
		6

l.	*s.*	*d.*
2	11	10
		7

l.	*s.*	*d.*
1	11	$0\frac{1}{2}$
		8

l.	*s.*	*d.*
2	0	7
		8

l.	*s.*	*d.*
1	11	10
		9

l.	*s.*	*d.*
1	1	$0\frac{1}{4}$
		10

l.	*s.*	*d.*
1	11	3
		11

l.	*s.*	*d.*
1	3	$10\frac{1}{2}$
		12

If 1 week's house-keeping cost 3*l.* 12*s.* $9\frac{1}{2}$*d.*, what would it cost for 2 weeks at the same rate?

Bought 3 hogsheads of French wine, at 21*l.* 2*s.* $1\frac{1}{4}$*d.* per hhd.: what did it cost?

Bought 3 tea-trays, price 1*s.* $1\frac{1}{4}$*d.* each: what did they cost?

Suppose 1 yard of superfine scarlet cost 1*l.* 2*s.* 4¼*d.*, what would 4 yards cost?

Bought 5 tooth-brushes, at 1*s.* 1½*d.* each, what did they cost?

If 1 pair of silk stockings cost 7*s.* 5½*d.*, what would 6 pair cost?

Sold 6 gallons of red port, at 13*s.* 10½*d.* per gallon: the price of the whole is required.

Six yards of rich brocade, at 14*s.* 1½*d.* per yard: the price of the whole is required.

Seven ton of Russian steel, at 24*l.* 18*s.* 2½*d.* per ton?

Bought seven pair of cotton stockings for 6*s.* 8½*d.* per pair, what did they cost?

Seven cwt. of Malaga raisins, at 12*s.* 4½*d.* per cwt.?

If 1 yard of Brussels carpet cost 11*s.* 0½*d.* what would 8 cost?

If 1 month's house-rent cost 10*s.* 8*d.*, what would 8 months cost?

Bought 9 yards of cambric muslin for 9*s.* 8½*d.* per yard, what did it cost?

Bought 9 small glasses, at 7½*d.* each, what did they cost?

Bought 9 gallons of palm sack, at 9*s.* 4½*d.* per gallon: what did it cost?

If 1 yard of broad cloth cost 8*s.* 6½*d.*, what would 10 yards cost?

Bought 11 pair of kid gloves, at 3*s.* 11¼*d.* per pair: what did they cost?

If 1 bushel of old wheat cost 7*s.* 7*d.*, what would 11 bushels cost?

Bought 11 steel thimbles for 6¼*d.* each: what did they cost?

If 1 gallon of Jamaica rum cost 7*s.* 3¼*d.*, what would 12 cost?

If 1 firkin of London porter cost 15*s.* 6¼*d.*, what would 12 cost?

If 1 pair of silk stockings cost 3*s.* 8¼*d.*, what would 12 pair cost?

If 1 lb. of imperial tea cost 1*l.* 7*s.* 4¼*d.*, what would 12 cost?

[When the multiplier exceeds 12, multiply successively by its component parts, instead of the whole number at once.]

Bought 14 couple of fat ducks, at 3*s.* 1¼*d.* the couple: what did they cost?

Suppose I buy 1 yard of Indian dimity for 5*s.* 9¼*d.*: what must I give for 15 yards at the same rate?

Bought 18 glass bottles at 3¼*d.* each: what did they cost?

If 1 dozen of mouse-traps cost 12*s.* 3¼*d.*, what would twenty dozen cost?

Bought 21 yards of Yorkshire camblets, price 4*s*. 3¼*d*. per yard; required the price of the whole.

If 1 gallon of old beer cost 3*s*. 4*d*., what would 25 cost?

If 1 pair of leather gloves cost 1*s*. 7*d*., what would 30 pair cost?

If 1 quarter of damaged oats cost 5*s*. 2¼*d*., what would 32 cost?

If 1 bushel of damaged barley cost 2*s*. 1*d*., what would 42 cost?

Bought 45 China oranges, at 1½*d*. each: what did they cost?

If 1 pair of brass joints cost 9½*d*., what would 56 pair cost?

If 1 pound of Gentian powder cost 5*s*. 5½*d*., what would 60 cost?

If 1 ell of fine muslin cost 1*s*. 7*d*., what would 66 ells cost?

If 1 chaldron of pit coal cost 10*s*. 4*d*., what would 70 cost?

If 1 yard of black velvet cost 1*s*. 2½*d*., what would be the price of 72 yards?

If 1 dozen of silver thimbles cost 3*s*. 1*d*., what would 80 dozen cost?

Bought 90 China oranges, at ½*d*. each; required the price of the whole?

Suppose 1 lb. of Cheshire cheese cost 3*s*. 0½*d*., what would 108 lbs. cost?

If 1 pair of leather gloves cost 4¼*d*., what would 120 pair cost?

If 1 ell of fine cotton cost 9½*d*., what would 132 cost?

[If the multiplier cannot be produced by the multiplication of simple numbers, take the lesser numbers nearest to it which can be so produced, and multiply by its parts, as before. Then multiply the multiplicand by the difference between this number and the multiplier, and add the product to that before found.]

Bought 39 pounds of French tobacco, at 8*s*. 4½*d*. per lb.: what did it cost?

Bought 69 cwt. of old iron, at 1*s*. 2½*d*. per cwt.: required the price of the whole.

Bought 1 ell of green baize for 1*s*. 3¼*d*: required the price of 83 ells?

If 1 lb. of black pepper cost 2*s*. 7*d*., what would be the price of 126 lbs.?

If 1 dozen of snuff boxes cost 2*s*. 7*d*., what would 95 dozen cost?

If 1 pair of cotton stockings cost 2*s*. 9*d*., what would 17 pair cost?

Bought 135 lbs. of red paint, price 2*s*. 4½*d*. per lb. : what did the whole cost ?

Bought 83 pen cases, at ½*d*. each : what did they come to ?

Bought 119 pair of shoe-buckles, at 4½*d*. per pair : what did they cost ?

Bought 1 score of old bottles for 3*s*. 2½*d*. : what would 125 score cost, at the same rate ?

Bought 93 ells of fine calico, at 1*s*. 1½*d*. per ell : what is the price of the whole quantity ?

If 1 pair of leather gloves cost 6*d*., what would 114 pair cost ?

Bought 122 packing needles, at ½*d*. each : what did they come to ?

Bought 137 yards of broad binding, at 7½*d*. per yard : required the price ?

Bought 1 ell of coarse diaper for 8½*d*. : what would 155 cost, at the same rate ?

Bought 151 dozen of small cups, at 1*s*. 7½*d*. per dozen : what did I give for the whole ?

If 1 lb. of roach alum cost 2*s*. 6½*d*. : what will 128 lbs. cost ?

Bought 23 cwt. of old rags, at 3*s*. 7½*d*. per cwt. : required the price.

Bought 78 lbs. of Turkey rhubarb, at 4*s*. 2½*d*. per lb. : what did it come to ?

[When the number exceeds 156, multiply by 10, and that product by 10, for 100; then multiply the price of 100 by the number of hundreds; the price of 10 by the number of tens; and the price of 1 by the unit figure.]

Bought 223 pair of cotton laces, at $2\frac{1}{4}d.$ per pair: what did they come to?

Bought 198 dozen of hen-eggs, at $1s.\ 2\frac{1}{4}d.$ per dozen: what did they cost?

If 1 yard of black fillet cost $4\frac{1}{4}d.$, what would 216 yards cost?

Bought 575 lbs. of raw sugar, price $6\frac{1}{2}d.$ per lb.: what did it cost?

Bought 194 barrels of new cyder, at $3s.\ 8\frac{1}{4}d.$ per barrel: what did it cost?

If 1 yard of common crape cost $4\frac{1}{4}d.$, what would 244 yards cost?

If 1 lb. of white ginger cost $1s.\ 3\frac{1}{4}d.$, what would 252 lbs. cost?

[If the number has $\frac{1}{4}$, $\frac{1}{2}$, or $\frac{3}{4}$, joined to it, add a fourth of the given price for $\frac{1}{4}$, half of the price for $\frac{1}{2}$, and a half and a quarter for $\frac{3}{4}$.]

If 1 yard of black velvet cost $19s.$, what would $4\frac{1}{4}$ yards cost?

Bought $1\frac{1}{4}$ cwt. of Malaga raisins, at $2l.\ 9s.\ 6d.$ per cwt.: required the price.

Bought 269¼ score of fine oranges, at 1*s.* 6*d.* per score: what did they come to?

Bought 61¼ bushels of damaged wheat for 1*s.* 5*d.* per bushel: what did the whole cost?

Bought 1 ell of green baize for 2*s.* 6½*d.*: required the price of 41¼ ells, at the same rate?

Bought 287¼ lbs. of raw sugar, price 1*s.* 1½*d.* per lb.: what did it come to?

Sold 649¼ dozen of hat buckles for 4½*d.* per dozen: what did they come to?

Sold 34½ cwt. of old iron, at 2*s.* 5*d.* per cwt.: what did it come to?

Bought 160¼ dozen of curtain rings, price 1*s.* 6½*d.* per dozen: what did they cost?

Bought 111¼ barrels of ginger-beer, at 6*s.* 6*d.* per barrel: required the price.

Bought 123½ bushels of foreign grain for 8½*d* per bushel: what did it cost?

Bought 61¼ lbs. of green paint, price 5*s.* 3½*d.* per lb.: what did it come to?

COMPOUND DIVISION.

COMPOUND DIVISION teaches to find how often one given number is contained in another of different denominations.

RULE.

Place the divisor and dividend as in Simple Division. Begin at the highest denomination of the dividend, which divide by the divisor, and write down the quotient.

If there be any remainder after this division, find how many integers of the next lower denomination it is equal to, and add them to the number, if any, which stands in that denomination.

Divide the number so found by the divisor, and write the quotient under its proper denomination.

Proceed in the same manner through all the denominations to the lowest; and the whole quotient, thus found, will be the answer required.

The method of proof is the same as in Simple Division.

	l.	s.	d.
2)	24	16	8
	12	8	4

	l.	s.	d.
3)	34	7	$9\frac{3}{4}$

	l.	s.	d.
4)	76	7	8

	l.	s.	d.
5)	76	7	11

	l.	s.	d.
6)	57	6	$10\frac{1}{2}$

	l.	s.	d.
7)	120	6	3

	l.	s.	d.
8)	106	19	4

	l.	s.	d.
9)	94	11	$10\frac{1}{2}$

	l.	s.	d.
10)	162	6	8

	l.	s.	d.
11)	157	11	6

	l.	s.	d.
12)	137	11	3

	l.	s.	d.
12)	217	14	0

If 2 pair of gold bracelets cost 4*l.* 9*s.* 6*d.*, what did one pair cost?

If 3 French Grammars cost 1*l.*, what did one cost?

If 3 dozen of silver cups cost 36*l.* 19*s.*, what was that per dozen?

Bought 4 young turkeys for 1*l.* 2*s.* 7*d.*: what was the price of one?

Four quarters of foreign wheat cost 33*l.* 9*s.* 8*d.*: the price per quarter is required.

If 5 combs of mustard-seed cost 21*l.* 16*s.* 8*d.*, what would 1 cost?

Bought 5 Dutch cheeses for 1*l.* 8*s.* 2¼*d.*: what did each cost?

Suppose 6 score of young pigs cost 121*l.* 12*s.* what did they cost per score?

Bought 6 silk purses for 1*l.* 7*s.* 4½*d.*: the price of one is required.

Six years' house-rent cost 73*l.* 12*s.*: what was the yearly rent?

Bought 6 yards of flowered silk, price 26*l.* 10*s.* 1½*d.* for the whole: required what it cost per yard?

If 7 straw hats cost 1*l.* 17*s.* 4*d.*, what was given per hat?

Bought 7 cwt. of raw sugar for 29*l.* 4*s.* 4¼*d.*: required the price per cwt.?

Seven weeks' labourers' wages cost 52*l.* 5*s* 11*d.*: what was the weekly expence?

Bought 8 pen-knives for 1*l.* 8*s.* 6*d.*: what did 1 cost?

Bought 8 cwt. of bees' wax, price 33*l.* 14*s.* 2*d.* for the whole quantity: what was given per cwt.?

Sold 8 quarters of foreign wheat for 66*l.* 19*s.* 10*d.*: required the price per quarter.

If 8 bushels of canary seed cost 34*l.* 10*s.* 8*d.*, what did 1 bushel cost?

Bought 8 spring locks, price 2*l.* 11*s.* 6*d.*: required the price of 1.

If 9 roods of grass land cost 362*l.* 8*s.*, what did it cost per rood?

If 9 silver spoons cost 2*l.* 18*s.* 6*d.*, what did one cost?

Nine work-boxes cost 1*l.* 19*s.* 9*d.*: the price of one is required.

Bought 10 cwt. of salt butter, price 42*l.* 5*s.* 2½*d.* for the whole: required the price per cwt.

Ten pencil-cases cost 3*l.* 4*s.* 2*d.*: what did one cost?

Eleven dozen of gold plates cost 134*l.* 9*s.* 6*d.*: what was that per dozen?

Eleven ivory rulers cost 3*l.* 0*s.* 6*d.*: what was that each?

If I buy 12 tons of any article for 242*l.* 3*s.* 9*d.*, what will it cost per ton?

Bought 12 oval dishes for 2*l.* 14*s.* 9*d.*, what did 1 cost?

[When the divisor exceeds 12, find what simple numbers, multiplied together, will produce it, and divide by them separately, as in Simple Division.]

If 15 cwt. of scented soap cost 65*l.* 10*s.*, what was that per cwt. ?

Sixteen Greek grammars cost 4*l.* 10*s.* 8*d.* : what was the price of 1 ?

Eighteen hogsheads of port wine cost 1137*l.* 18*s.* : what was that per hogshead ?

If 21 gilt seals cost 4*l.* 12*s.* 9*d.*, what did 1 cost ?

If 22 months' house-keeping cost 94*l.* 2*s.* 10*d.*, what was the expence per month ?

Bought 24 cwt. of refined sugar, price 104*l.* 18*s.* 6*d.* : what did I give per cwt. ?

If 25 cotton shawls cost 8*l.* 2*s.* 6*d.*, what was the price of one ?

Bought 28 boys' hats, price 6*l.* 1*s.* 11*d.* : what was the price per hat ?

If 30 dozen of mahogany chairs cost 372*l.* 15*s.*, what was the price of 1 dozen ?

Bought 36 lamp-glasses for 8*l.* 7*s.* 3*d.* : what did 1 cost ?

Bought 40 tea-trays, price 6*l.* 17*s.* 6*d.* : what did 1 cost ?

If 48 hogsheads of Jamaica rum cost 3041*l.* 8*s.*, what would be the price of 1 hogshead?

If 54 German flutes cost 17*l.* 11*s.*, what did 1 cost?

Bought 56 hair brushes, price 12*l.* 7*s.* 10*d.*: what did 1 cost?

Bought 60 prayer-books for 19*l.* 8*s.* 9*d.*: what was that each?

Sold 66 gallons of French brandy for 285*l.* 13*s.* 1½*d.*: what was that per gallon?

Sold 72 young turkeys for 20*l.* 6*s.* 6*d.*: how much was that each?

Suppose 77 comb of hemp-seed cost 324*l.* 13*s.* 8*d.*; what was the price of a comb?

Bought 80 Pope's Homers for 22*l.* 5*s.*: required the price of 1.

If 84 Simpson's Euclids cost 36*l.* 1*s.*: what was the price of one?

Bought 90 silk purses, price 20*l.* 6*s.* 10½*d.* for the whole: what did 1 cost?

Bought 96 pocket-books for 31*l.* 2*s.*: required the price of 1.

Bought 108 straw hats for 29*l.* 2*s.* 9*d.*: what was the price of 1?

If 120 dozen of gold seals cost 1466*l.* 10*s.*: what was the price per dozen?

Bought 132 watch chains for 36*l.* 14*s.* 3*d.*: required the price of one?

If 144 tons of white soap cost 2918*l.* 8*s.*: what was the price of a ton?

[When the divisor cannot be produced by the multiplication of small numbers, divide by the whole divisor at once, after the manner of Long Division.]

If 13 hogsheads of Lisbon wine cost 823*l.* 2*s.* 4*d.*: what did it cost per hogshead?

Bought 19 sheep-skins for 5*l.* 4*s.* 1½*d.*: what did they cost per skin?

Bought 23 Roman Histories, price 6*l.* 12*s.* 8½*d.*: required the price of one?

If 31 tea-caddies cost 5*l.* 8*s.* 6*d.*, what did one cost?

Bought 34 dozen of pocket knives, price 419*l.* 1*s.* 8½*d.*: what did I give per dozen?

If 39 weeks' household expences cost 291*l.* 17*s.*, what was the weekly expence?

If 41 tons of Russian hemp cost 835*l.* 0*s.* 8*d.*, required the price per ton?

Bought 47 old volumes for 8*l.* 11*s.* 4½*d.*: what was that per volume?

Bought 53 China basins for 14*l.* 14*s.* 9¾*d.*: what did one cost?

If 65 waistcoat pieces cost 30*l.* 17*s.* 6*d.*, what is the price of one?

If 79 months' labourers' wages cost 353*l.* 3*s.* 11*d.*, what was that per month?

Bought 86 cwt. of Cheshire cheese for 380*l.* 16*s.* 4½*d.*: required the price per cwt.?

Bought 92 bent combs, price 20*l.* 8*s.* 3*d.*: what did one cost?

If 105 tons of black rosin cost 2128*l.* 8*s.* 9*d.*, what was the price per ton?

Bought 116 silk hats for 25*l.* 9*s.* 11*d.*: what was the price of one?

If 126 score of Norfolk sheep cost 2566*l.* 14*s.* 6*d.*, what was the price of one?

Bought 176 English Grammars for 68*l.* 0*s.* 4*d.*: what was the price of one?

If 134 cwt. of Dutch cheese cost 572*l.* 17*s.*, what did one cwt. cost?

Bought 168 hat brushes for 30*l.* 12*s.* 6*d.*: required the price of one?

Suppose 189 pair of gold buckles cost 421*l.* 6*s.* 3*d.*: what would be the price per pair?

Bought 192 looking-glasses for 73*l.* 8*s.*: what was that each?

Bought 196 lasts of damaged corn for 2032*l.* 9*s.* 7*d.*: what was given per last?

Suppose 212 cwt. of any article cost 885*l.* 19*s.* 8*d.*: what would be the price per cwt.?

Bought 248 plated tankards for 83*l.* 8*s.* 10*d.*: what did one cost?

If 255 common hats cost 80*l.* 15*s.*, what was the price of one?

Bought 276 cwt. of Irish bacon for 1179*l.* 12*s.* 3*d.*: required the price per cwt.?

If 285 cotton shawls cost 92*l.* 12*s.* 6*d.*, what did one cost?

Suppose 294 hogsheads of red port cost 18571*l.*; what would be the price per hhd.?

If 348 soldiers' hats cost 145*l.*, what was the price per hat?

Bought 464 white feathers, price 131*l.* 19*s.*: what did one cost?

Bought 824 quarters of new wheat, price 673*3l.* 12*s.* 6*d.*: what was that per quarter?

Bought 948 sheep-skins for 259*l.* 14*s.* 3*d.*: the price of one is required.

A library, containing 1236 old books cost 211*l.* 3*s.*: required the price per volume, supposing them to be of equal value?

WEIGHTS AND MEASURES.

ADDITION.

Troy Weight.

lb.	oz.	dwt.	gr.
24	6	13	11
81	0	8	20
19	10	13	22
74	8	6	23
24	4	10	7
42	3	5	4

lb.	oz.	dwt.	gr.
12	4	13	4
12	1	13	22
11	5	14	23
43	9	16	14
32	6	14	13
54	1	4	10

lb.	oz.	dwt.	gr.
20	6	3	4
35	3	2	14
73	1	11	13
10	2	10	10
81	4	15	11
31	5	2	11

lb.	oz.	dwt.	gr
12	1	3	1
23	2	10	12
11	9	13	8
41	4	1	14
23	7	14	14
26	2	5	5

Avoirdupois Weight.

tons.	cwt.	qrs.	lb.
21	2	2	13
22	13	1	24
11	10	3	9
56	7	1	10
44	11	0	1
11	2	3	10
53	14	2	2
31	10	1	6

cwt.	qr.	lb.	oz.
12	2	16	10
32	1	11	6
11	3	6	14
22	1	13	4
10	0	7	7
10	3	13	4
12	2	3	13
32	1	5	4

tons.	cwt.	qr.	lb.
21	5	1	4
24	19	2	15
11	13	1	13
33	15	1	4
94	8	3	10
11	11	1	11
23	11	2	3
34	6	2	8

cwt.	qr.	lb.	oz.
10	2	6	1
12	1	13	12
21	1	12	11
13	2	3	7
19	1	11	6
10	3	11	13
22	2	2	15
13	1	4	7

Apothecaries' Weight.

lb	℥	ʒ	℈	℥	ʒ	℈	gr.
11	3	4	1	11	1	1	3
12	5	7	2	22	4	2	12
11	11	2	1	11	1	1	4
17	1	6	0	13	3	2	11
25	3	2	2	21	1	0	2
21	1	2	0	10	0	2	16
12	0	3	1	21	3	2	15
24	1	1	1	11	6	1	2

Long Measure.

yds.	feet	in.	b.-c.	leagues.	m.	fur.	poles.
13	2	1	1	31	1	5	21
22	2	3	2	33	2	3	23
4	1	11	1	93	1	7	26
43	2	9	2	59	2	4	13
2	0	10	1	41	0	6	22
15	1	5	1	38	2	3	23
13	2	7	2	32	1	6	13
31	2	2	1	52	1	2	5

Square Measure

acres.	roods.	perches.
22	2	31
35	3	33
49	2	29
22	3	18
56	1	23
14	0	32
20	3	25
46	1	6

yds.	ft.	
32	2	3
36	8	143
11	1	21
33	0	102
88	3	30
96	1	11
34	2	125
45	6	140

Cloth Measure.

yds.	qrs.	na.
23	1	2
24	2	3
32	3	2
76	2	1
13	3	0
20	1	3
31	2	3
14	2	1

ells.	qrs.	na.
43	2	2
23	2	1
76	4	0
98	3	2
82	1	3
73	2	0
45	1	2
31	2	1

Wine Measure.

hhd.	gal.	qts.	pts.
24	21	1	1
33	60	2	1
76	22	3	1
18	53	1	0
27	10	3	1
30	29	1	0
13	12	0	1
38	16	2	0

tons.	p.	hhd.	gal.
41	0	1	10
34	0	1	61
16	1	1	37
96	1	0	61
83	0	1	56
95	1	0	15
72	1	1	22
35	1	0	40

Beer Measure.

bar.	kil.	fir.	gal.
44	1	0	3
23	0	1	8
74	1	0	7
97	0	1	5
93	0	0	6
28	0	1	3
82	1	1	7
53	0	1	5

bar.	gal.	qts.	pts.
21	15	2	0
12	14	3	1
60	8	2	1
34	14	2	1
46	3	3	1
12	11	1	1
34	18	2	1
20	16	1	1

Dry Measure.

pks.	gal.	qts.	pts.
21	0	1	1
46	1	3	1
13	1	1	0
67	0	3	1
12	1	1	0
39	1	0	1
19	1	2	1
62	1	2	0

lasts.	qrs.	bus.	pks.
21	3	1	2
33	5	3	3
14	7	7	2
89	6	5	0
18	4	0	3
10	2	1	2
33	3	3	2
16	6	6	1

Time.

days.	ho.	min.	sec.
42	10	5	46
72	23	43	54
94	14	17	13
33	8	24	9
90	12	10	24
56	10	11	39
52	8	12	41
45	9	41	8

yrs.	mo.	wks.	days.
21	3	2	2
47	2	3	3
23	1	2	4
38	10	0	5
15	3	3	4
16	0	2	2
59	4	3	3
75	2	1	3

SUBTRACTION.

Troy Weight.

From 10 lb. 0 oz. 3 dwt. 23 gr., take 2 lb. 6 oz. 8 dwt. 14 gr.

From 12 lb. 10 oz. 4 dwt. 14 gr., take 8 lb. 1 oz. 12 dwt. 2 gr.

Bought 13 lb. 5 oz. 16 dwt. 20 gr. of silver, and sold 3 lb. 2 oz. 18 dwt. 14 gr.: what remained?

What is the difference between 10 lb. 10 oz. 1 dwt. 23 gr. and 19 lb. 2 oz. 18 dwt. 7 gr.?

Avoirdupois Weight.

From 35 tons. 6 cwt. 1 qr. 20 lb., take 18 tons. 2 cwt. 3 qrs. 21 lb.

Take 6 tons. 1 cwt. 0 qr. 18 lb., from 24 tons. 3 cwt. 3 qrs. 16 lb.

What is the difference between 12 tons. 6 cwt. 2 qrs. 26 lb., and 31 tons. 8 cwt. 2 qrs. 23 lb.?

A grocer bought goods weighing qrs. 26 lb. 23 oz. dr. 3 5: what was the weight after having sold from the same qrs. 8 lb. 12 oz. 12 dr. 11?

Apothecaries Weight.

From ℔ 10 ℥ 3 ʒ 3 ℈ 1 gr. 2, take ℔ 4 ℥ 8 ʒ 1 ℈ 1 gr. 4.

What is the difference between ℔ 2 ℥ 11 ʒ 7 ℈ 0 gr. 4, and ℔ 9 ℥ 6 ʒ 1 ℈ 2 gr. 3?

Long Measure.

From ft. 22 in. 1 b.c. 1, take ft. 10 in. 11 b.c. 2.

From yd. 8 ft. 1 in. 2, take yd. 6 ft. 1 in. 4.

What is the difference between miles. 3 fur. 7 poles. 36, and miles. 5 fur. 7 poles. 29?

A person had to perform a journey of lea. 18 m. 2 fur. 5, after travelling lea. 16 m. 2 fur. 7, how far had he to go?

Square Measure.

From a. r. p. 5 3 6, take a. r. p. 3 3 9.

An estate contained a. r. p. 36 1 32, from which was sold a. r. p. 33 2 34: what quantity remained?

Cloth Measure.

From yd. qr. na. 21 3 0, take yd. qr. na. 19 3 3.

From E. ells. qr. na. 7 1 1, take E. ells. qr. n. 2 4 2.

From Fr. ells. qr. na. 8 1 2, sold Fr. ells. qr. na. 3 5 0: what remained?

If Fl. ells. qr. na. 7 2 3 be cut from a piece containing Fl. ells. qr. na. 10 1 2, how much will be lost?

Wine Measure.

From hhd. gal. qts. 84 1 2, take hhd. gal. qts. 21 62 3.

Take hhd. gal. qts. 30 5 3 from hhd. gal. qt. 90 9 0.

What is the difference between 31 1 1 31, (tuns p. hhd. gal.)
and 33 1 1 30? (tuns p. hhd. gal.)

A merchant's stock of wine was 92 2 0 1: (hhd. gal. qt. pt.)
after selling 31 61 3: what remained? (hhd. gal. qts.)

Beer Measure.

From 12 4 1, take 4 2 2. (fir. gal. qt. — fir. gal. qts.)

What is the difference between 3 0 1 6, (bar. kil. fir. gal.)
and 5 0 1 5? (bar. kil. fir. gal.)

After selling 12 33 3 1 of beer, what (bar. gal. qts. pt.)
remained, the stock being before 46 1 1? (bar. gal. qt.)

Dry Measure.

Take 5 9 4 2 from 9 6 6 1. (last qrs. bus. pks. — last qrs. bus. pk.)

What is the difference between 16 1 2 1, (last qr. bus. pk.)
and 11 4 7 3? (last qrs. bus. pk.)

Time.

What is the difference between 9 yrs. 11 mo. 0 w. 4 d. 20 h. and 19 yrs. 2 mo. 2 w. 4 d. 14 h.?

If 3 w. 5 da. 6 ho. 47 min. 56 sec. be subtracted from 6 w. 4 da. 2 h. 29 min. 15 sec. what will be the remainder?

MULTIPLICATION.

Troy Weight.

Multiply 2 lb. 5 oz. 18 dwt. 11 gr. by 3.

Multiply 1 lb. 10 oz. 7 dwt. 11 gr. by 5.

What is the weight in pounds, ounces, &c. of eleven silver cups, each weighing 6 oz. 3 dwt. 1 gr.?

What is the weight of 8 pieces of silver, each weighing 1 lb. 0 oz. 12 dwt. 1 gr.?

Avoirdupois Weight.

Multiply qr. lb. oz. dr. 1 23 7 1 by 10.

Multiply tons cwt. qr. lb. 3 0 1 23 by 6.

What is the weight of the whole, if the eleventh part weighs qr. lbs. oz. dr. 1 26 2 5?

Apothecaries' Weight.

Multiply ℔ ℥ ʒ ℈ gr. 2 9 4 2 19 by 2.

If the third part is ℔ ℥ ʒ ℈ gr. 2 2 0 2 13, what is the weight of the whole?

Long Measure.

What will be the product in leagues, miles, &c. of miles fur. 2 7, multiplied by 2?

What will be the length in yards, &c. ft. in. 1 7, multiplied by 5?

A person walked seven times over a piece

fur. poles.
of ground, measuring 6 11: how far did he walk?

Square Measure.

yd. ft. in.
Multiply 1 3 71 by 4.

Bought six pieces of land, each measuring
r. p.
1 33: what was the whole quantity in acres?

Cloth Measure.

What will be the product in yards, &c. of
qr. na.
1 2, multiplied by 7?

E. ells qr. na.
Multiply 1 2 1 by 3.

Bought five pieces of cloth, each measuring
qr. na.
2 3; what was the whole quantity in yards, &c.?

Wine Measure.

hhd. gal. qts.
Multiply 30 3 3 by 2.

hhd. gal. qt.
Multiply 8 54 1 by 7.

What is the tenth part of 171 tons. 14 cwt. 3 qrs. 18 lb.?

What quantity multiplied by 12 will make 174 qr. 2 lb. 2 oz. 8 dr.?

Apothecaries' Weight.

Divide 39 lb. 2 ℥ 5 ʒ 2 ℈ 6 gr. by 7.

What is the fifth part of 32 lb. 7 ℥ 5 ʒ 0 ℈ 15 gr.?

Long Measure.

Divide 22 m. 3 fur. 10 po. by 6.

Divide 23 lea. 1 m. 7 fur. by 9.

Find the eleventh part of 21 yds. 1 ft. 2 in.

Find the twelfth part of 122 ft. 4 in.

Square Measure.

Divide 20 a. 3 r. 34 p. by 6.

What is the fifth part of 9 a. 3 r. 25 p.?

If 32 a. 3 r. 16 p. of land be equally divided among 12 persons, what will be the share of each?

Cloth Measure.

Divide 30 yd. 3 qr. 3 na. by 9.

Divide 35 E. ells by 10.

What quantity of cloth is contained exactly 11 times in 38 Fr. ells. 5 qr. 3 na.?

What quantity of cloth multiplied by 12 will make 22 Flemish ells?

Wine Measure.

Divide 496 hhd. 14 gal. by 8.

Divide 549 hhd. 22 gal. 2 qr. by 9.

What quantity of wine is contained exactly 10 times in 19 tons. 1 p. 1 hhd. 53 gal.?

What quantity of wine 12 times taken will
hhd. gal. qts.
make 720 40 2?

Beer Measure.

bar. kil. fir. gal.
Find the eleventh part of 21 1 0 7.

What quantity of beer 7 times taken will
bar. gal. qts. pt.
make 231 23 2 1?

fir. gal. qts.
If 80 0 2 be equally divided among 11 persons, what will be the quantity in each share?

Dry Measure.

What quantity of corn is contained exactly
lasts. qrs. bus. pks.
5 times in 18 6 0 3?

bus. pk. gal. qt.
Divide 24 1 0 1 into 7 equal parts.

Time.

yrs. mo. wks. da. ho.
Divide 81 9 2 4 8 by 8.

w. da. h. min. sec.
Find the ninth part of 17 5 0 20 42.

BILLS OF PARCELS.

Mr. William Simpson,

Bought of John Cole.

		s.	d.	£	s.	d.
14 spelling-books,	at	0	7¼			
7 large slates,	at	0	9½			
6 silver pens,	at	1	0½			
10 English grammars,	at	0	9½			
10 pocket-books,	at	0	7¼			
1 pair of steel compasses,	at	11	3			

Mr. Thomas Smith,

Bought of Daniel Hurst.

		s.	d.	£	s.	d
9 gallons of palm sack,	at	9	4½			
90 dozen of China oranges,	at	2	8¾			
5¼ cwt. of Malaga raisins,	at	16	6			
34½ gallons of old beer,	at	12	5			
37¼ barrels of spruce beer,	at	19	6			
1 hhd. of white vinegar, price		£1	19	2¼		

Mr. GEORGE WILSON,

Bought of SAMUEL CURTIS.

		s.	d.	£	s.	d.
10 pair of ladies' shoes,	at	4	7¼			
6 pair of red morocco,	at	7	3¼			
10 pair of black Spanish,	at	4	6¼			
1 sheep skin,		3	5			
411 yards of black binding,	at	0	2½			
1 dozen of lamb skins,	at	19	8½			

Mr. EDWARD WATSON,

Bought of CHARLES HILL.

		s.	d.	£	s.	d.
6 yards of Brussels lace,	at	14	8¾			
10 pair of kid gloves,	at	4	4¼			
10 gallons of white vinegar,	at	8	6¾			
1 Dutch cheese		8	4			
9 yards of French cambric,	at	9	7½			
1 lb. of green tea,	at	12	9½			

Mr. Simon Lowe,

Bought of Joseph Martin.

		£	s.	d.	£	s.	d.
6 tons of bay salt,	at	3	7	2¼			
3 cwt. of brown soap,	at	1	8	5½			
10 paint brushes,	at	0	0	6¼			
14 gallons of lamp oil,	at	0	6	0½			
6 gallons of spruce beer,	at	0	1	4¼			
1 barrel of olive oil,	at	3	0	3			

Mr. James Hardy,

Bought of Robert Bell.

		s.	d.	£	s.	d.
7 pair of ear-rings,	at	6	2½			
10 pencil-cases,	at	0	7¼			
10 pair of gilt bracelets,	at	4	5¼			
2 gilt seals,	at	2	2½			
3 pair of gold buckles,	at	14	10½			
1 dozen of table spoons, price		£5	1	8¼		

Mr. STEPHEN GRAY,

Bought of EDMUND REED,

		s.	d.	£	s.	d.
50 yards of London print,	at	1	8¼			
11 pair of kid gloves,	at	3	11½			
6 yards of fine scarlet,	at	14	1¼			
6 leather caps,	at	1	2¼			
10 ells of blue silk,	at	10	5¼			
1 lb. of scented soap,	at	1	0			

Mr. MICHAEL JONES,

Bought of WILLIAM NEWCOME.

		s.	d.	£	s.	d.
5 spring locks,	at	1	3¼			
6 iron shovels,	at	0	9¼			
10 brass weights,	at	0	6¼			
14 pewter measures,	at	0	5¼			
6 brass taps,	at	0	10¼			
1 pair of copper scales,	at	17	9			

Mr Isaac Hall,

Bought of John Webb.

	£	s.	d.	£	s.	d.
Bill delivered,	74	6	10½			
519 lbs. of iron wire, at	0	7	¾			
14¼ cwt. of old lead. at	5	9				
45 dozen of stove brushes, at per doz.	5	5	¼			
42 tons of old iron, at	9	7	¼			
388¼ lbs. of old copper, at	0	10				
1 gross of steel thimbles, at	18	6	¼			

Mr. William Robins,

Bought of Henry Holmes.

	£	s.	d.	£	s.	d
Sundry articles,	9	0	0			
98¼ dozen of cedar pencils, each at	0	2	¼			
6 small maps, at	0	10	¼			
7 spelling-books, at	1	2	¼			
14 pair of brass compasses, at	3	3	¼			
1 quire of writing paper, at	2	2	¼			

Mr. EDWARD STEEL,

Bought of THOMAS HOWARD.

	£	s.	d.	£	s.	d.
New grand piano,	114	0	0			
42 dozen of covered strings, at per doz.		5	10¼			
17 select airs, at		0	4½			
5 Scotch songs, at		1	3½			
411 ounces of steel wire, at		0	9½			
1 gross of brass pins, at		19	6¼			

Mr. JOHN WILES,

Bought of CHARLES PRATT.

	£	s.	d.	£	s.	d.
Bill delivered,	50	10	2½			
750 quires of thin post, at		0	7¼			
10 Murray's grammars, at		0	9½			
37¼ dozen of steel pens, each at		0	6½			
6 English readers, at		1	3½			
32¼ dozen of small primers, each at		0	7½			
One hundred crow quills, at		3	6½			

RUSSELL'S APPENDIX

TO

WALKER'S ARITHMETIC.

REDUCTION.

Definition. — REDUCTION is the method of converting money, weight, or measure, from one name or denomination to another, without altering its intrinsic value; and it is either *Descending*, *Ascending*, or *Mixed*.

REDUCTION DESCENDING.

Definition. — Reduction Descending is that which is employed in reducing higher names to lower ones; as, to bring pounds to shillings, pence, or farthings; or cwts. to qrs., lbs., or oz., &c.

Rule 1.—Multiply by as many of the next lower name* as make one of that which is given, adding the number of this name (if any) mentally.

2. — Then multiply this last denomination by as many of the following lower name as make one of it, adding, mentally, as before; and so on, till the required denomination be produced.†

* I would recommend the pupil to write the name of each denomination as he produces it.

† If the given number be a *simple one*, it may sometimes be more easily brought to the lowest denomination by one multiplication; and even a compound quantity may be reduced by this method, by adding the value of the lower denominations to the product of the highest name, multiplied by as many of the lower as make one of it.

EXAMPLE I.

Reduce *l.* *s.* *d.*
3 14 6¼ to farthings.
20

74 shill.
12

894 pence.
4

3578 farth.—Ans.

EXAMPLE II.

Reduce 12 cwt. 1 qr. 10 lb. to pounds.

Method 1.

cwt. qr. lb.
12 1 10
4

49 qrs.
28

402
98

1382 lbs.—Ans.

or,

Method 2.

cwt.
12
112

144
12

38 = 1 qr. 10 lb.

1382 lbs. as before.

Elucidation.—The first example requires no more explanation than that pounds, multiplied by 20, produce shillings; shillings by 12, give pence; and pence by 4, give farthings.

In the first method of the second Example, cwts. are multiplied by 4 to produce qrs., and qrs. by 28 to give lbs.

Method 2.—Th done agreeably to the note at bottom of page 101.

REDUCTION ASCENDING.

Definition.—Reduction Ascending is that which

is employed in bringing lower names to higher ones; as to bring farthings to pence, shillings, or pounds; or oz. to lbs., qrs., or cwts., &c.

Rule 1.—Divide the given denomination by as many of its name as make one of the next higher.

2.—Then divide this quotient by as many of its name as make one of the next higher, and so on till you produce the required denomination.*

EXAMPLE I.

In 3578 farthings, how many pounds?

4) 3578 farth.

12) 894½ pence.

20) 74*s.* 6*d.*

£3 14 6½.—Ans.

EXAMPLE II.

In 1382 lbs. how many cwts.?

Method 1.

28 { 4) 1382 lbs.
   { 7) 345 2

4) 49 qrs. and 2 × 4 + 2 = 10 lbs.†

12 cwt. 1 qr. 10 lbs.—Ans.

* The lowest denominations may, sometimes, be brought to the highest by one division more expedit ously than by the common method.

† To find the true remainder when you divide by *three* component parts; multiply the second by the first divisor, adding the first remainder (if any); to this add the product of the last remainder, multiplied by the 1st and 2d divisors.

Or,

Method 2.

112) 1382 (12 cwt. 1 qr. 10 lb., as before.
112

262
224

38 = 1 qr. 10 lb.

This case, being the reverse of the preceding, requires no elucidation.

MIXED REDUCTION.

Definition. — Mixed Reduction is the method of finding how many of a required denomination are equivalent to a given number of another denomination, when one of the higher name does not contain an *exact* number of the lower; as, to bring guineas to pounds; or English ells to yards, &c.

Rule 1.—Bring one of the given name and one of that which is required into the same denomination.

2. — Consider, from the nature of the question, whether the answer should be more or less than the given number.

3.—If more, multiply the given quantity by the greater, and divide by the less; but, if less, the contrary.*

Example.

In 500 guineas how many pounds?

* The operation in Mixed Reduction may often be abridged by adding or subtracting a part of the given number, thus: guineas may be brought to pounds, by adding $\frac{1}{10}$; pounds to guineas by subtracting $\frac{1}{21}$; Eng. ells to yds. by adding $\frac{1}{4}$; and yds. to English ells by subtracting $\frac{1}{5}$, &c.

Method 1.	or,	*Method* 2.
500 gs.		$\frac{1}{20}$) 500
21		25
20) 10,500 sh.		£525 as before.
£525 ans.		

ELUCIDATION.—*Method* 1. A guinea contains 21sh. and a pound 20 sh.; and, since a guinea is 1 sh. more than a pound, any given number of guineas must consequently contain a greater number of pounds; 21 has therefore been employed as a multiplier, and 20 as a divisor.

Method 2.—As a guinea is a pound and $\frac{1}{20}$ of a pound, $\frac{1}{20}$ of any given number of guineas added to itself, must produce the number of pounds contained therein.

EXAMPLES.

1. *Money.*

1. In £18, how many farthings?—Ans. 17280.
2. In £250 13*s*. 9½. how many farthings?—Ans. 240661.
3. In £172 16*s*. 8½*d*. how many half-pence?—Ans. 82961.
4. In £34 17*s*. 9*d*. how many pence?—Ans. 8373.
5. In £17 13*s*. 6*d*. how many two-pences?—Ans. 2121.
6. In £13 18*s*. 9*d*. how many three-pences?—Ans. 1115.
7. In £17 19*s*, 8*d*. how many groats?—Ans. 1079.
8. In £19 11*s*. 6*d*. how many sixpences?—Ans. 783.
9. In 17280 farthings, how many pounds?—Ans. £18.
10. In 240661 farthings, how many pounds?—Ans. £250, 13*s*. 9½*d*.
11. In 82961 half-pence, how many pounds?—Ans. £172 16*s*. 8½*d*.
12. 8373 pence, how many pounds?—Ans. £34 17*s*. 9*d*.
13. In 2121 two-pences, how many pounds?—Ans. £17 13*s*. 6*d*.

14. In 1115 three-pences, how many pounds?—Ans. £13'18*s.* 9*d.*

15. In 1079 groats, how many pounds?—Ans. £17 19*s.* 8*d.*

16. In 783 sixpences, how many pounds?—Ans. £19 1*s.* 6*d.*

17. In 45 guineas, how many pence?—Ans. 11340.

18. In 11340 pence, how many guineas?—Ans. 45.

19. In £39, how many crowns and half-crowns?—Ans. 156 crowns, and 312 half-crowns.

20. In 40 guineas, how many sixpences?—Ans. 1680.

21. In 400 seven-shilling pieces, how many farthings?—Ans. 134400.

22. In 134400 farthings, how many seven-shilling pieces?—Ans. 400.

23. In £37,254, how many guineas?—Ans. 35480.

24. In 35480 guineas, how many pounds?—Ans. 37254.

25. How many crowns in £172. 10*s.*?—Ans. 690.

26. How many moidores in 7668 guineas?—Ans. 5964.

2. *Troy Weight.*

27. Reduce 25 lb. 11 oz. 21 gr. to grains?—Ans. 149301.

28. In 149301 grains, how many lbs.?—Ans. 25 lb. 11 oz. 21 gr.

29. In 7 ingots of silver, each weighing 23 lb. 5 oz. 7 dwt., how many grains?—Ans. 945336.

30. In 945336 grains, how many ingots, each containing 23 lb. 5 oz. 7 dwt.?—Ans. 7.

31. A gentleman sent a tankard to his silversmith weighing 75 oz. 12 dwt., and desired to have in return as many spoons of 2 oz. 16 dwt. each, which could be produced from it; what number should he receive?—Ans. 27.

3. *Avoirdupois Weight.*

32. In 17 tons, how many drams?—Ans. 9748480.

33. In 9748480 drams, how many tons?—Ans. 17.

34. In 13 cwt. 1 qr. 17 lb. how many lb. and oz.?—Ans. 24016 oz.

35. In 48 cwt. 1 lb. how many drams?—Ans. 1376512.

36. In 123456789 drams, how many tons?—Ans. 215 tons, 5 cwts. 3 qr. 9 lb. 1 oz. 5 dr.

37. How many parcels of sugar, each containing 16 lb. 2 oz. are there in 16 cwt. 1 qr. 2lb. 2 oz.?—Ans. 113.

4. *Apothecaries' Weight.*

38. Reduce 27 lb. 7 oz. 2 dr. 1 scr. 2 gr. into grains. —Ans. 159022.

39. In 159022 grs. how many lb.?—Ans. 27 lb. 7 oz. 2 dr. 1 scr. 2 gr.

5. *Cloth Measure.*

40. Reduce 12 yds. 3 qrs. 2 na. 1½ in. into inches?—Ans. 464½.

41. In 464½ in.* how many yards?—Ans. 12 yds. 3 qrs. 2 na. 1½ in.

42. Reduce 543 ells. 4 qrs. Eng. to quarters and nails. —Ans. 10876 nails.

43. In 10876 na. how many Eng. ells?—Ans. 543 ells 4 qrs.

44. In 329 ells. 5 qrs. French, how many quarters?—Ans. 1979.

45. In 1769 ells. Eng. how many yards?—Ans. 2200.

46. In 736 ells French, how many ells Flem.?—Ans. 1472.

47. In 839 yards, how many ells Eng.?—Ans. 671 ells E. 1 qr.

48. In 17 pieces of cloth, each 27 ells Flem. how many yards?—Ans. 344 yds. 1 qr.

5. *Long Measure.*

49. In 128 miles how many furlongs, poles, and yards?—Ans. 225280 yards.

50. In 225280 yards how many poles†, furlongs, and miles?—Ans. 128 miles.

51. How many barley-corns would it require to reach round a circle as great as it is at the equator of the terraqueous globe, allowing 69½ miles to a degree? —Ans. 4755801600.

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* For the division by 2½, multiply both divisor and dividend by the under figure of the fraction, and afterwards divide in the usual way; and, to find the proper remainder, divide what is over by the lower figure of the fraction.

† For the division by 5½ and 30½, in this question, proceed in the same way as directed for the division by 2½.

52. How many times will a wheel, which is 2½ yards in circumference, turn round in running from London to York, which is a distance of 198 miles?—Ans. 126720.

6. *Land Measure.*

53. Reduce 3695 ac. 3 ro. 11 po. to poles?—Ans. 591331.

54. In 591331 poles, how many acres?—Ans. 3695 ac. 3 ro. 11 po.

55. In 17887430 yards,* how many roods?—Ans. 14783 roods.

56. In 18 ac. 39 po. how many yards?—Ans. 882994.

7. *Wine Measure.*

57. What number of gallons are there in 5 tuns of port wine?—Ans. 1260.

58. In 10080 pints, how many tuns?—Ans. 5.

59. In 32 hhds. how many gallons and quarts.—Ans. 8064 quarts.

60. In 16128 pints, how many pipes?—Ans. 16.

61. In 1 gallon, how many gills?—Ans. 32.

8. *Ale, or Beer Measure.*†

62. In 90 barrels of beer, how many pints?—Ans. 25920.

63. In 25920 pints of ale, how many barrels?—Ans. 90.

64. In 100 hhds., how many barrels?—Ans. 150.

65. In 17 firkins of ale, how many gills?—Ans. 4896.

9. *Dry Measure.*

66. In 312 ch. 6 bush. 2 pks. 1 gal. 5 pts. how many pints?—Ans. 719277.

67. In 768164 gallons, how many quarters?—Ans. 12002 qrs. 4 bush. 2 pks.

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* For the division by 5½ and 30¼, in this question, see note to question 41, in the preceding page.

† There is now no distinction whatever between the ale and beer firkin, for they each contain 9 gallons, and there are consequently 36 gallons to the barrel. By a lawsuit lately decided, a publican recovered 2 gallons per barrel from a brewer, who had vended by the barrel of 34 gallons during the whole period of their mutual transactions.

68. In 1 wey or load, how many quarts?—Ans. 1280.

10. *Measure of Time.*

69. In 14 years, of 52¼ weeks each, how many weeks, days, hours, minutes and seconds?—Ans. 441504000 seconds.

70. In 441504000 seconds, how many minutes, hours, days, weeks, and years?—Ans. 14.

71. From January 7th till October 9th, 1823, how many days?—Ans. 275.

72. From October 9th, 1823, till March 7th, 1824, how many days?—Ans. 150.

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73. How many shillings, half-crowns, crowns, and half-guineas, and of each an equal number, are there in £475?—Ans. 500.

74. How much oftener than a large wheel of 2¼ yds. in circumference, will a small one, which is 1½ yards, turn round in running a distance of 10 miles?—Ans. 5333⅓ times.

75. How many minutes have elapsed since the birth of our Saviour to the end of the year 1823, supposing each year to be 365¼ days?—Ans. 958825080.

76. How long would a person be in counting a million of sovereigns, supposing he reckoned 100 in a minute, and continued at it 10 hours every day?—Ans. 16 days, 6 hours, 40 minutes.

ARITHMETICAL FRACTIONS.

Definition.—An Arithmetical Fraction is a part or parts of a unit, or of any integer or whole, and is expressed by two numbers, the one above and the other below a horizontal line, as ½ ⅓, &c.

The number below the line is called the *denominator*, and it shews into how many parts the unit, integer, or whole, is divided.

The number above the line is called the *numerator*, and shews the number of parts of the unit, integer, or whole, which the fraction contains:—thus, in the faction $\frac{3}{4}$ the denominator 4 indicates that the unit is divided into 4 equal parts, and the numerator 3 shews that 3 of those equal parts are to be taken for the value of the fraction.

There are four different kinds of Arithmetical Fractions, viz. *proper*, *improper*, *compound*, and *complex*.

1. A *proper fraction* is one whose value is *less* than unity, and has always the numerator *less* than the denominator, as $\frac{1}{2}$, $\frac{2}{3}$, &c.

2. An *improper fraction* is one that is either *equal to* or *greater* than unity; and is known by its numerator being either *equal to* or *greater than* its denominator, as $\frac{4}{4}$*, or $\frac{12}{7}$, &c.

3. A *compound fraction* is the fraction of a fraction, and is known by two or more fractions being linked together by the preposition *of*, as $\frac{1}{2}$ of $\frac{7}{8}$, of $\frac{3}{9}$, &c.

4. A *complex fraction* is such as has a fraction in *either*, or *both*, of its members, as $\frac{3\frac{1}{2}}{5}$, $\frac{7}{4\frac{1}{2}}$, $\frac{\frac{4}{5}}{9}$, or $\frac{7\frac{1}{2}}{21\frac{1}{2}}$, &c.

Note 1. A mixed number is composed of a whole number and a fraction, as $3\frac{1}{2}$, or $12\frac{7}{8}$, &c.

2. An integer may be expressed fractionally by placing 1 below it as a denominator; thus, 4 may be expressed by $\frac{4}{1}$.†

REDUCTION OF ARITHMETICAL FRACTIONS.

Definition.—Reduction of Arithmetical Fractions

* When the numerator and denominator are equal, the fraction is equal to unity, or 1.

† When we wish to have any other denominator besides 1, we multiply the integer by it; thus, $4 = \frac{4}{2}$, or $\frac{8}{2}$, or $\frac{12}{3}$, or $\frac{16}{4}$, &c.

is the method of bringing them from one form into another, in order to prepare them for the several operations in Addition, Subtraction, Multiplication, and Division.

Case 1. To reduce mixed numbers to equivalent improper fractions.

Rule.—Multiply the integer by the denominator of the fraction, and to the product add the numerator; this sum, written above the denominator, will give the required fraction.

EXAMPLE.

Reduce $3\frac{3}{4}$ to an improper or simple fraction

$$3\frac{3}{4}$$
$$4$$
$$\overline{15} \therefore 3\frac{3}{4} = \frac{15}{4}. \quad \text{Ans.}$$

Elucidation.—Here $3\frac{3}{4}$ have been multiplied by 4, which produces 15. It is evident that it has thereby become 4 times its given value; but, if this product be made, the numerator of a fraction, with 4 as its denominator, it is obvious that the fraction will just be equal to what it was as a mixed number; for any quantity multiplied and divided by the same number can neither be increased nor diminished; hence the rule is manifest. See Elucidation to Case 5.

Case 2.—To reduce improper fractions to whole or mixed numbers.

Rule.—Divide the numerator by the denominator; the quotient will be the whole number; and, should there be any remainder, under it subscribe the denominator for the remaining fraction.

EXAMPLE.

Reduce $\frac{15}{4}$ to an equivalent whole, or mixed number.

$$4 \overline{)\, 15}$$
$$\overline{3\frac{3}{4}}. \quad \text{Ans.}$$

Elucidation.—Here 15 has been divided by 4, which gives $3\frac{3}{4}$ as the equivalent mixed number for $\frac{15}{4}$; which

is evidently correct, from this—that, as there are four fourths (4/4)* in unity or 1, the number of times that four fourths are contained in fifteen fourths must give the number of *ones*, and the remainder the number of fourths.

Case 3.—To reduce fractions to their lowest terms.—This may be done either by inspection, that is, by dividing both members of the fraction by any number or numbers, at pleasure, which those members contain an exact number of times; or by the following

Rule.—Divide the greater term by the less; the less by the remainder; the first remainder by the second; and the second by the third; and so on till there be no remainder: the last divisor will be the greatest common measure of the fraction; therefore, the numerator and denominator divided by this number, will produce the lowest terms of the given fraction.

Note 1.—If the unit's place of both members of a fraction be an even number, the whole fraction is divisible by 2.

2. If the unit's and ten's places of both members be divisible by 4, the whole fraction is divisible by 4.

3. If the unit's, ten's, and hundred's places can be measured by 8, the fraction is divisible by 8.

4. If the unit's place be a cypher or 5, the fraction is divisible by 5.

5. If the numerator and denominator end with one, two, three cyphers, &c. the fraction is divisible by 10, 100, 1000, &c.

6. If 9 or 3 measure the sum of the significant figures in both members, the fraction is divisible by 9 or 3.

7. If the units and tens be divisible by 25, the whole is divisible by 25.

8. If the unit's place of any number be even, and the sum of all the digits divisible by 6, the whole is divisible by 6.

9. A number is divisible by 11, when the sum of the 1st, 3d, 5th, &c. digits is equal to the sum of the 2d, 4th, 6th, &c.

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* See note at bottom of page 110.

10. The greatest common measure* of any fraction is the greatest number that can possibly be found to divide the numerator and denominator without a remainder.

EXAMPLE I.

Reduce $\frac{112}{124}$ to its lowest terms.

Method 1.

$\frac{112}{124} = \frac{14}{18} = \frac{1}{2}$.—Ans.

Or, *Method* 2.

$\frac{112}{124} = \frac{28}{31} = \frac{7}{14} = \frac{1}{2}$, as before.

Elucidation.—In method 1, as the unit's, ten's, and hundred's places, in both numbers, are divisible by 8, 8 has been employed as a divisor, agreeably to Note 3, which produces $\frac{14}{18}$; and both numbers of this divided by 7 gives $\frac{1}{2}$, which is the lowest term of the fraction. Method 2 is managed in its first and second steps agreeably to Note 2, and afterwards by employing 7 as a common measure.

EXAMPLE II.

Reduce $\frac{1008}{1296}$ to its lowest terms.

1008) 1296 (1
1008

288) 1008 (3
864

144) 288 (2
288

Hence 144 is the greatest common measure, and both members of the fraction $\frac{1008}{1296}$ divided by 144 will produce $\frac{7}{9}$, as the lowest terms of it.

Remark.—That 144 is the greatest common divisor of 1008 and 1296, is evident, from this—that the num-

* When 1 is found to be the greatest common measure, the fraction is already in its lowest terms, and is therefore said to be *incommensurable.*

ber which is contained in two other numbers must be contained in their difference; so that, 144 must be an aliquot part of 288 and 144; but the greatest number contained in 144 is 144 itself; therefore 144 is the greatest common measure.

Note.—As it sometimes may be necessary to find the greatest common measure of more than two numbers, the following rule will be found to answer that purpose; viz. find the greatest common measure of two of them, as before; and of that common measure, and one of the other numbers, and so on, through all the numbers to the last, and the greatest common measure last found will be the answer.

EXAMPLE.

Required the greatest common measure of 918, 1998, and 552.

First 522) 918 (1
522
396) 522 (1
396
126) 396 (3
378
18) 126 (7
126

Then 18) 1998 (111
18
19
18
18
18

Therefore 18 is the greatest common measure of the three numbers, 918, 1998, and 522.

Case 4. To reduce compound fractions to simple ones.

Rule 1. If part of the compound fraction be an integer, a mixed number, or a complex fraction, reduce it to its proper terms.

Note 1. Numbers that are common to both numerator and denominator may be cancelled, and the other parts abbreviated by the notes in page 112; afterwards multiply the remaining numerators together, for the numerator of the required simple fraction; and the denominators for the required denominator.

EXAMPLE.

Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{6}{7}$ to a simple fraction.

$$\frac{\cancel{2} \times \cancel{3} \times 5 \times \cancel{6}}{\cancel{3} \times 4 \times \cancel{6} \times 7} = \frac{5}{2 \times 7} = \frac{5}{14}. \quad \text{Ans.}$$

$\cancel{2}$

Note 2. That a compound fraction, or a fraction of a fraction, can be represented by a simple one is evident; since part of a part must be equal to some part of a whole.

Suppose the simple fractional expression were required for $\frac{2}{3}$ of $\frac{3}{4}$ of a yard of cloth. It is evident, that $\frac{1}{3}$ of $\frac{3}{4}$, or of 3 qrs., is just 1 qr. of a yard; therefore, $\frac{2}{3}$ of 3 qrs. must be twice 1 qr. which is just half a yard, so that $\frac{2}{3}$ of $\frac{3}{4}$ may be more simply represented by $\frac{1}{2}$.

Note 3. In the above example small horizontal lines are drawn over the numerators and under the denominators that are cancelled.

Case 5. To reduce fractions to a common denominator.

Rule. Multiply each numerator by all the denominators except its own, for a new numerator; then multiply all the denominators together for a common denominator.

Note. If there be an integer or mixed number, a compound, or complex fraction, in the question, it must be reduced to the form of a simple fraction before proceeding as directed in the rule.

EXAMPLE I.

Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$, to a common denominator.

$$\left.\begin{array}{l} 1 \times 3 \times 4 \times 5 = 60 \\ 2 \times 2 \times 4 \times 5 = 80 \\ 3 \times 2 \times 3 \times 5 = 90 \\ 4 \times 2 \times 3 \times 4 = 96 \end{array}\right\} \text{new numerators for} \left\{\begin{array}{l} \frac{1}{2}. \\ \frac{2}{3}. \\ \frac{3}{4}. \\ \frac{4}{5}. \end{array}\right.$$

$2 \times 3 \times 4 \times 5 = 120$ the common denominator.

$$\left.\begin{array}{rl} \therefore \frac{1}{2} & = \frac{60}{120} \\ \frac{2}{3} & = \frac{80}{120} \\ \frac{3}{4} & = \frac{90}{120} \\ \text{And } \frac{4}{5} & = \frac{96}{120} \end{array}\right\} \text{Ans.}$$

Elucidation. In reducing fractions to a common denominator, we change their expression without altering their value: thus, in the above example, we find $\frac{1}{2}$ equal to $\frac{60}{120}$, which expression is precisely the value of $\frac{1}{2}$; for, suppose that we divide 10*s.* into 120 equal parts, 60 of those equal parts would just be $\frac{1}{2}$ of 10*s.*; consequently, $\frac{60}{120}$ are equal to $\frac{1}{2}$, which serves to prove that, when both members of a fraction are multiplied by the same number, the value will remain unaltered. The following order of the preceding example will, from the slightest inspection, show, that the numerator and denominator of each fraction is multiplied by precisely the same numbers, and, consequently, neither increased nor diminished in value.

$$\begin{array}{l|l} \frac{1}{2} & \times \frac{3}{3} \times \frac{4}{4} \times \frac{5}{5} = \frac{60}{120}. \\ \hline \frac{2}{3} & \times \frac{2}{2} \times \frac{4}{4} \times \frac{5}{5} = \frac{80}{120}. \\ \hline \frac{3}{4} & \times \frac{2}{2} \times \frac{3}{3} \times \frac{5}{5} = \frac{90}{120}. \\ \hline \frac{4}{5} & \times \frac{2}{2} \times \frac{3}{3} \times \frac{4}{4} = \frac{96}{120}. \end{array}$$

It may be asked why we bring fractions to a common denominator? the reason is simply this: when they are of different denominators, they are entirely dissimilar, and therefore cannot be incorporated with each other.

In the above example, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$, cannot be easily added together; but, in adding their equivalents, $\frac{60}{120}$, $\frac{80}{120}$, $\frac{90}{120}$, and $\frac{96}{120}$, there is no difficulty; hence, the rules for Addition and Subtraction are manifest.

EXAMPLE II.

Reduce $\frac{3}{4}$, $1\frac{1}{9}$, and 3, to a common denominator.

First, $1\frac{1}{9} = \frac{10}{9}$ per *Case* 1 and $3 = \frac{3}{1}$, per *Note* 2, p. 110.

Now we have $\frac{3}{4}$, $\frac{10}{9}$, and $\frac{3}{1}$, to reduce to a common denominator.

$$\left.\begin{array}{l} \text{Then } 3 \times 9 \times 1 = 27 \\ 10 \times 4 \times 1 = 40 \\ 3 \times 4 \times 9 = 108 \end{array}\right\} \text{Num.} \qquad \left.\begin{array}{l} \therefore \frac{3}{4} = \frac{27}{36} \\ 1\frac{1}{9} = \frac{40}{36} \\ \text{And } 3 = \frac{108}{36} \end{array}\right\} \text{Ans.}$$

$4 \times 9 \times 1 = 36$ Den.

Case 6. To find the least common multiple* of two or more given numbers or denominators.

Rule. Write the given numbers with a period between each; then divide as many of them as possible by the smallest number which can be chosen for that purpose; place the undivided numbers in the same line with the several quotients, and divide again by any other small number; and so on, till the different quotients be prime or incomposite numbers:† the product of the divisors and last quotients will be the multiple required.

Note. 1. If the numbers admit of no common measure or divisor at first, their product is the common multiple.

2. If one or more of the given numbers be multiples of any of the others, reject those numbers of which they are the multiples.

EXAMPLE.

What is the least common multiple of 5, 6, 3, and 4.

$$2)\,5\,.\,6\,.\,\overline{3}\,.\,4 \text{ per } \textit{Note } 2.$$

$$5\,.\,3\,.\quad .\,2$$

$$\therefore\ 2 \times 5 \times 3 \times 2 = 60.\text{—Ans.}$$

Case 7. To reduce fractions to their least common denominator.

Rule. If necessary, reduce to simple fractions, as before; then find the least common multiple of all the denominators, which will be the least common denominator: this, multiplied by the numerator and divided by the denominator of each fraction, will give the numerator of each respective fraction, under which the least common denominator must be placed.

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* The *common multiple* of two or more given numbers is that which contains them an exact number of times, and the *least common multiple* is that which contains them the least possible number of times.

† A prime or incomposite number is that which is not divisible exactly, but by itself and unity.

EXAMPLE.

Reduce $\frac{5}{12}$, $\frac{4}{9}$, $\frac{3}{4}$, $\frac{2}{3}$, and $\frac{5}{8}$, to fractions having the least common denominator.

First, 3) 12 . 9 . 4 . 3 . 8

4) 4 . 3 8

1 . 3 2

$\therefore 3 \times 4 \times 3 \times 2 = 72$, the least com. denom

Then, $\frac{72 \times 5}{12} = 30$ new num. for $\frac{5}{12} \therefore \frac{5}{12} = \frac{30}{72}$

$\frac{72 \times 4}{9} = 32$ ditto for $\frac{4}{9} \therefore \frac{4}{9} = \frac{32}{72}$

$\frac{72 \times 3}{4} = 54$ ditto for $\frac{3}{4} \therefore \frac{3}{4} = \frac{54}{72}$

$\frac{72 \times 2}{3} = 48$ ditto for $\frac{2}{3} \therefore \frac{2}{3} = \frac{48}{72}$

And $\frac{72 \times 5}{8} = 45$ ditto for $\frac{5}{8} \therefore \frac{5}{8} = \frac{45}{72}$

In finding the least common multiple, we have rejected, agreeably to note 2 of this case, 4 and 3, as 12 is a multiple of both.

Case 8. To reduce any fraction to another of a given denominator.

Rule. Multiply the given numerator by the new denominator; this divided by the denominator of the given fraction, will give the numerator of the required fraction.

EXAMPLE.

Reduce $\frac{3}{8}$ to a fraction of the same value, whose denominator shall be 64.

$$\frac{3 \times 64}{8} = 24 \text{ new num.}$$

$$\therefore \frac{3}{8} = \frac{24}{64}.\text{—Ans.}$$

Case 9. To reduce a fraction to another of a given numerator.

Rule. Multiply the denominator of the given fraction by the new numerator; this product divided by the numerator of the given fraction will be the denominator required.

EXAMPLE.

Reduce $\frac{3}{8}$ to a fraction of the same value, whose numerator shall be 12.

$$\frac{8 \times 12}{3} = 32, \text{ new den.}$$

$$\therefore \frac{3}{8} = \frac{12}{32}.\text{—Ans.}$$

Case 10. To reduce complex fractions to simple ones.

Rule. Reduce the integral or mixed members, if any, to simple fractions; the product of the upper and under figures will be the required numerator; likewise, the product of the two middle figures will be the required denominator.

EXAMPLE.

What is the simple fraction for $\frac{7}{1\frac{3}{4}}$?

First, $7 = \frac{7}{1}$, per *Note* 2, p. 110, and $1\frac{3}{4} = \frac{7}{4}$, per *Case* 1.

Then $\frac{\frac{7}{1}}{\frac{7}{4}} = \frac{4 \times 7}{7 \times 1} = \frac{28}{7}$ or $\frac{4}{1}$.—Ans.

Case 11. To reduce money, weights, and measures, into fractions.

Rule. Reduce the given money, weight, or measure, into the lowest name mentioned for the numerator, and reduce the given integer into the same name for the denominator; this fraction reduced to its lowest terms will be the answer.

EXAMPLE.

What part of a pound are 7*s*. 6*d*.

Method 1.	or,	*Method* 2.
First 7*s*. 6*d*. = 90 pen.		7*s*. 6*d*. = 15 sixp.
And £1 = 240 pen.		And 20*s*. = 40 sixp.
$\frac{90}{240} = \frac{9}{24} = \frac{3}{8}$.—Ans.		$\therefore \frac{15}{40} = \frac{3}{8}$.—Ans.

Case 12. To find the value or quantity of a fraction.

Rule. Consider the numerator as so many of the name or denomination of which the fraction is a part, which divide by the denominator precisely in the same way as directed in Compound Division.

EXAMPLE.

What is the value of $\frac{3}{4}$ of a pound sterling?

Here we consider the numerator 3 as £3,
And £3 ÷ 8 = 7*s.* 6*d.*—Ans.

Case 13. To reduce fractions from one denominator to another equivalent in value.

Rule. If the reduction be from a higher to a lower denomination, multiply the numerator by as many of the lower as make one of the higher; but, if from a lower to a higher, multiply the denominator by as many of the lower as make one of the higher. This fraction, in its lowest terms, will be the answer.

EXAMPLE I.

Reduce $\frac{4}{9}$ of a farthing to the fraction of a pound.

$$\frac{4}{9} \times 960 = \frac{4}{3840} = \frac{1}{2160}. \quad \text{Ans.}$$

EXAMPLE II.

Reduce $\frac{1}{2160}$ of a pound to the fraction of a farthing.

$$\frac{1}{2160} \times 960 = \frac{960}{2160} = \frac{96}{216} = \frac{8}{18} = \frac{4}{9}. \quad \text{Ans.}$$

Note. If we have to reduce from a denomination which is not contained an exact integral number of times in the denomination which is required, or the contrary, the best way is, first to reduce a unit of each into a name common to them both; then, if from the nature of the question, the required fraction should be greater than that which is given, we multiply the numerator by the greater, and the denominator by the less; but, if the fraction ought to be less than the given one, the contrary; thus, let us reduce $\frac{3}{4}$ of a gui :ea to the fraction of a pound.

A guinea = 21*s*. and a pound = 20*s*.

Now as it requires a greater portion of a pound than $\frac{4}{5}$ to equal the same portion of a guinea, which is greater than a pound, we therefore multiply the numerator of the given fraction by 21, and the denominator by 20. Thus, $\frac{4}{5} \times \frac{21}{20} = \frac{84}{100} = \frac{21}{25}$.—Ans.

EXAMPLES TO CASE 1.

1. Reduce $7\frac{2}{3}$ to a simple fraction.—Ans. $\frac{23}{3}$.
2. Reduce $15\frac{4}{9}$ to a simple fraction.—Ans. $\frac{139}{9}$.
3. Reduce $19\frac{6}{13}$ to a simple fraction.—Ans. $\frac{253}{13}$.
4. Reduce $5\frac{3}{16}$ to a simple fraction.—Ans. $\frac{83}{16}$.
5. Reduce $18\frac{11}{12}$ to a simple fraction.—Ans. $\frac{227}{12}$.
6. Reduce $874\frac{5}{7}$ to a simple fraction.—Ans. $\frac{6123}{7}$.
7. Reduce $300\frac{5}{79}$ to a simple fraction.—Ans. $\frac{23715}{79}$.
8. Reduce $967\frac{11}{38}$ to a simple fraction.—Ans. $\frac{36757}{38}$.
9. Reduce $112\frac{3}{43}$ to a simple fraction.—Ans. $\frac{4717}{43}$.
10. Reduce $1\frac{2002}{1001}$ to a simple fraction.—Ans. $\frac{2004}{1001}$.
11. Reduce $2\frac{702}{701}$ to a simple fraction.—Ans. $\frac{2103}{701}$.
12. Reduce $19\frac{1}{9}$ to a simple fraction.—Ans. $\frac{172}{9}$.

EXAMPLES TO CASE 2.

13. Reduce $\frac{23}{3}$ to a whole or mixed number.—Ans. $7\frac{2}{3}$.
14. Reduce $\frac{139}{9}$ to a whole or mixed number.—Ans. $15\frac{4}{9}$.
15. Reduce $\frac{253}{13}$ to a whole or mixed number.—Ans. $19\frac{6}{13}$.
16. Reduce $\frac{83}{16}$ to a whole or mixed number.—Ans. $5\frac{3}{16}$.
17. Reduce $\frac{227}{12}$ to a whole or mixed number.—Ans. $18\frac{11}{12}$.
18. Reduce $\frac{6123}{7}$ to a whole or mixed number.—Ans. $874\frac{5}{7}$.

19. Reduce $\frac{23718}{79}$ to a whole or mixed number.—Ans. $300\frac{18}{79}$.

20. Reduce $\frac{26787}{38}$ to a whole or mixed number.—Ans. $967\frac{11}{38}$.

21. Reduce $\frac{4717}{42}$ to a whole or mixed number.—Ans. $112\frac{13}{42}$.

22. Reduce $\frac{2001}{1001}$ to a whole or mixed number.—Ans. $1\frac{1000}{1001}$.

23. Reduce $\frac{2102}{701}$ to a whole or mixed number.—Ans. $2\frac{700}{701}$.

24. Reduce $\frac{172}{9}$ to a whole or mixed number.—Ans. $19\frac{1}{9}$.

EXAMPLES TO CASE 3

1. *By Inspection.*

25. Reduce $\frac{49}{98}$, $\frac{69}{78}$, $\frac{130}{195}$, $\frac{15}{25}$, $\frac{138}{190}$, and $\frac{173}{182}$, to their lowest terms.—Ans. $\frac{1}{2}$, $\frac{6}{7}$, $\frac{13}{19}$, $\frac{3}{5}$, $\frac{27}{38}$, and $\frac{43}{46}$.

26. Reduce $\frac{208}{884}$, $\frac{2544}{2912}$, $\frac{3720}{5676}$, and $\frac{784}{952}$, to their lowest terms.—Ans. $\frac{52}{171}$, $\frac{91}{104}$, $\frac{310}{473}$, and $\frac{14}{17}$.

27. Reduce $\frac{210000}{211500}$, $\frac{70100}{571100}$, and $\frac{131000}{271000}$, to their lowest terms.—Ans. $\frac{8}{9}$, $\frac{701}{5711}$, and $\frac{210}{453}$.

2. *By the Rule.*

28. Reduce $\frac{2240}{7216}$, $\frac{1743}{4950}$, $\frac{621}{846}$, and $\frac{715}{1209}$, to their lowest terms.—Ans. $\frac{285}{901}$, $\frac{581}{2830}$, $\frac{69}{94}$, and $\frac{286}{603}$.

29. Reduce $\frac{918}{1998}$, $\frac{2158}{4672}$, and $\frac{492216}{1364976}$, to their lowest terms.—Ans. $\frac{17}{37}$, $\frac{1573}{2336}$, and $\frac{3}{8}$.

30. Reduce $\frac{1274}{1348}$, $\frac{1881}{2792}$, and $\frac{1792}{1848}$, to their lowest terms.—Ans. $\frac{637}{674}$, $\frac{241}{1396}$, and $\frac{896}{924}$.

31. Is $\frac{76327356}{189967329}$ a commensurable or an incommensurable fraction?

EXAMPLES TO CASE 4.

32. Reduce $\frac{2}{3}$ of $\frac{7}{8}$ of $\frac{1}{2}$ to a simple fraction.—Ans. $\frac{7}{24}$.

33. Reduce $\frac{3}{4}$ of $\frac{1}{2}$ of $\frac{4}{5}$ of $\frac{5}{6}$ to a simple fraction.—Ans. $\frac{1}{2}$.

34. Reduce $\frac{4}{9}$ of $\frac{5}{7}$ of $\frac{6}{11}$ of $\frac{9}{13}$ to a simple fraction. Ans. $\frac{120}{1001}$.

35. Reduce $\frac{5}{6}$ of $\frac{1}{17}$ of $11\frac{1}{2}$ of $14\frac{2}{7}$ to a simple fraction.—Ans. $\frac{275}{1428}$.

36. Reduce $\frac{1}{2}$ of $\frac{2}{3}$ of $6\frac{1}{4}$ to a simple fraction.—Ans. $\frac{5}{12}$.

37.* Reduce $\frac{1}{3}$ of $\frac{2\frac{1}{2}}{5\frac{1}{4}}$ of $\frac{1}{7}$ of $\frac{\frac{1}{2}}{\frac{3}{7}}$ to a simple fraction. —Ans. $\frac{1}{126}$.

EXAMPLES TO CASE 5.†

38. Reduce $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{5}{6}$, to a common denominator.—Ans. $\frac{240}{360}$, $\frac{270}{360}$, $\frac{288}{360}$, and $\frac{300}{360}$.

39. Reduce $\frac{7}{8}$, $\frac{5}{9}$, $\frac{6}{11}$, and $\frac{9}{13}$, to a common denominator.—Ans. $\frac{9009}{10296}$, $\frac{5720}{10296}$, $\frac{6048}{10296}$, and $\frac{6336}{10296}$.

40. Reduce $\frac{3}{7}$, $\frac{4}{11}$, $\frac{6}{13}$, and $\frac{1}{20}$, to a common denominator.—Ans. $\frac{3300}{10010}$, $\frac{7280}{10010}$, $\frac{9240}{10010}$, and $\frac{1001}{10010}$.

41. Reduce $11\frac{1}{4}$, $3\frac{1}{7}$, and 5, to a common denominator.—Ans. $\frac{315}{28}$, $\frac{88}{28}$, and $\frac{140}{28}$.

42. Reduce $\frac{2}{7}$ of $\frac{1}{2}$, $\frac{3}{5}$ of $\frac{1}{4}$ of $\frac{2}{7}$, and $5\frac{1}{6}$, to a common denominator.—Ans. $\frac{1440}{7840}$, $\frac{504}{7840}$, and $\frac{45010}{7840}$.

43.‡ Reduce $\frac{2\frac{1}{6}}{3\frac{1}{3}}$, $\frac{3}{\frac{7}{9}}$, and $\frac{\frac{2}{3}}{5\frac{1}{6}}$, to a common denominator.—Ans. $\frac{146025}{243040}$, $\frac{237840}{243040}$, and $\frac{18816}{243040}$.

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* This question may be omitted till the pupil has studied Case 10.

† See Case 7.

‡ This question may be omitted till the pupil has studied Case 10.

EXAMPLES TO CASE 6.

44. What is the least common multiple of 4, 8, 12, 16, and 20?—Ans. 240.

45. What is the least common multiple of 3, 5, 7, and 15?—Ans. 105.

46. What is the least common multiple of 18, 27, 14, and 9?—Ans. 378.

47. What is the least common multiple of 8, 7, 6, 5, 4, and 3?—Ans. 840.

48. What is the least common multiple of the nine digits?—Ans. 2520.

EXAMPLES TO CASE 7.*

49. Reduce $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{5}{6}$, to their least common denominator.—Ans. $\frac{40}{60}$, $\frac{45}{60}$, $\frac{48}{60}$, and $\frac{50}{60}$.

50. Reduce $\frac{7}{8}$, $\frac{8}{9}$, $\frac{6}{11}$, and $\frac{5}{13}$, to their least common denominator.—Ans. $\frac{9009}{10296}$, $\frac{8720}{10296}$, $\frac{5616}{10296}$, and $\frac{6336}{10296}$.

51. Reduce $\frac{3}{7}$, $\frac{4}{11}$, $\frac{6}{13}$, and $\frac{12}{15}$, to their least common denominator.—Ans. $\frac{8580}{20020}$, $\frac{7280}{20020}$, $\frac{9240}{20020}$, and $\frac{16016}{20020}$.

52. Reduce $11\frac{1}{2}$, $3\frac{1}{7}$, and 5, to their least common denominator.—Ans. $\frac{315}{28}$, $\frac{88}{28}$, and $\frac{140}{28}$.

53. Reduce $\frac{2}{7}$ of $\frac{1}{2}$, $\frac{3}{5}$ of $\frac{1}{4}$ of $\frac{3}{7}$, and $5\frac{1}{4}$, to their least common denominator.—Ans. $\frac{30}{140}$, $\frac{9}{140}$, and $\frac{105}{140}$.

54.† Reduce $\frac{2\frac{1}{3}}{3\frac{1}{2}}$, $\frac{3}{\frac{7}{8}}$, and $\frac{\frac{3}{5}}{5\frac{1}{6}}$, to their least common denominator.—Ans. $\frac{23716}{34720}$, $\frac{133920}{34720}$, and $\frac{2688}{34720}$.

EXAMPLES TO CASE 8.

55. Reduce $\frac{2}{3}$ to a fraction of equal value, whose denominator shall be 18.—Ans. $\frac{12}{18}$.

* The examples in this Case are precisely the same as those given in Case 5.

† The pupil may omit this question till he has studied *Case* 10.

56. Reduce $\frac{5}{7}$ to a fraction of equal value, whose denominator shall be 42.—Ans. $\frac{30}{42}$.

57. Reduce $\frac{13}{17}$ to a fraction of equal value, whose denominator shall be 51.—Ans. $\frac{39}{51}$.

EXAMPLES TO CASE 9.

58 Reduce $\frac{3}{5}$ to an equal fraction, whose numerator shall be 12.—Ans. $\frac{12}{20}$.

59. Reduce $\frac{5}{7}$ to an equal fraction, whose numerator shall be 30.—Ans. $\frac{30}{42}$.

60. Reduce $\frac{13}{17}$ to an equal fraction, whose numerator shall be 36.—Ans. $\frac{36}{51}$.

EXAMPLES TO CASE 10.*

61. What is the simple fraction for $\frac{9}{1\frac{1}{2}}$?—Ans. $\frac{6}{1}$.

62. What is the simple fraction for $\frac{1\frac{1}{2}}{9}$?—Ans. $\frac{1}{6}$.

63. What is the simple fraction for $\frac{7\frac{1}{2}}{8\frac{1}{7}}$?—Ans. $\frac{3}{2}\frac{1}{2}\frac{7}{7}$.

64. What is the simple fraction for $\frac{\frac{3}{5}}{3}$?—Ans. $\frac{1}{5}$.

65. What is the simple expression for $\frac{3}{\frac{5}{7}}$?—Ans. $\frac{4}{1}$.

EXAMPLES TO CASE 11.

66. What parts of a pound are 6*s*. 8*d*., 13*s*. 4*d*., and 16*s*. 8*d*.?—Ans. $\frac{1}{3}$, $\frac{2}{3}$, and $\frac{5}{6}$.

67. What parts of a shilling are $11\frac{1}{2}d.$, $9\frac{1}{4}d.$, and $4\frac{7}{11}d.$?—Ans. $\frac{2}{2}\frac{3}{4}$, $\frac{1}{1}\frac{3}{2}$, and $\frac{1}{4}\frac{7}{4}$.

68. What part of a ton are 17 cwt. 3 qrs. 14 lb.?—Ans. $\frac{1}{1}\frac{4}{2}\frac{3}{0}$.

69. What part of a cwt. are 15 lb. 13 oz. 12 dr.?—Ans. $\frac{7}{1}\frac{4}{0}\frac{5}{2}\frac{5}{4}$.

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* No other variety in complex fractions can happen than are exhibited in the five examples of this Case.

70. What part of a lb. are 3 oz. 15 dwt. 20 gr.?—Ans. $\frac{95}{288}$.

71. What part of a lb. are 1 oz. 7 dr. 2 scr. 19 gr.?—Ans. $\frac{959}{5760}$.

72. What part of a yard are 2 qrs. 1 na. 1¼ in.?—Ans. $\frac{43}{72}$.

73. What part of an ell Eng. are 3 na. 1¼ in.?—Ans. $\frac{17}{20}$.

74. What part of a mile is a barley-corn?—Ans. $\frac{1}{190080}$.

75. What part of a solar year, which is 365 da. 5 ho. 48 min. 48 sec., are 300 da. 17 ho. 10 min. 11 sec.?—Ans. $\frac{24328}{41552911}$.

EXAMPLES TO CASE 12.

76. What are the respective values of $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{5}{6}$, of a pound sterling?—Ans. 6*s.* 8*d.* 13*s.* 4*d.* and 16*s.* 8*d.*

77. What are the values of $\frac{11}{20}$, $\frac{13}{16}$, and $\frac{17}{144}$, of a shilling?—Ans. 11½*d.*, 9¾*d.*, and $4\frac{7}{11}d.$

78. What is the value of $\frac{143}{160}$ of a ton?—Ans. 17 cwt. 3 qrs. 14 lbs.

79. What is the value of $\frac{145}{1024}$ of a cwt.?—Ans. 15 lb. 13 oz. 12 dr.

80. What is the value of $\frac{95}{288}$ of a lb. troy?—Ans. 3 oz. 15 dwt. 20 gr.

81. What is the value of $\frac{959}{5760}$ of a lb. apoth.?—Ans. 1 oz. 7 dr. 2 scr. 19 gr.

82. What is the value of $\frac{43}{72}$ of a yard?—Ans. 2 qrs. 1 na. 1¼ in.

83. What is the value of $\frac{17}{20}$ of an ell Eng.?—Ans. 3 na. 1¼ in.

84. What is the value of $\frac{1}{190080}$ of a mile?—Ans. A barley-corn.

85. What is the value of $\frac{24328}{41552911}$ of a solar year of 365 da. 5 ho. 48 m. 48 sec.?—Ans. 300 da. 17 ho. 10 min. 11 sec.

EXAMPLES TO CASE 13.

86. Reduce $\frac{1}{3}$ of a pound to the fraction of a penny.—Ans. $\frac{240}{3}$ or $\frac{80}{1}$.

87. Reduce $\frac{3}{4}$ of a shilling to the fraction of a farthing.—Ans. $\frac{36}{1}$.

88. Reduce $\frac{11}{12}$ of a cwt. to the fraction of a lb.—Ans. $\frac{308}{3}$.

89. Reduce $\frac{5}{7}$ of a lb. troy to the fraction of a dwt.—Ans. $\frac{720}{7}$.

90. Reduce $\frac{1}{3}$ of $\frac{1}{4}$ of $\frac{1}{5}$ of a yard to the fraction of an inch.—Ans. $\frac{9}{5}$.

91. Reduce $\frac{\frac{1}{7}}{\frac{1}{8}}$ of $\frac{3\frac{1}{12}}{2\frac{7}{8}}$ of a mile to the fraction of a yard.—Ans. $\frac{86592}{35}$.

92. Reduce $\frac{80}{1}$ of a penny to the fraction of a pound.—Ans. $\frac{1}{3}$.

93. Reduce $\frac{36}{1}$ of a farthing to the fraction of a shilling.—Ans. $\frac{3}{4}$.

94. Reduce $\frac{308}{3}$ of a lb. to the fraction of a cwt.—Ans. $\frac{11}{12}$.

95. Reduce $\frac{720}{7}$ of a dwt. to the fraction of a lb.—Ans. $\frac{5}{7}$.

96. Reduce $\frac{9}{5}$ of an inch to the fraction of a yard.—Ans. $\frac{1}{20}$.

97. Reduce $\frac{86592}{35}$ of a yard to the fraction of a mile.—Ans. $\frac{33}{1760}$.

EXAMPLES TO THE NOTE.

98. What part of a pound is $\frac{7}{8}$ of a guinea?—Ans. $\frac{49}{80}$.

99. What part of a guinea is $\frac{49}{80}$ of a pound?—Ans. $\frac{7}{8}$.

100. What part of a crown is $\frac{3}{8}$ of a moidore?—Ans. $\frac{11}{20}$.

101. What part of a moidore is $\frac{11}{20}$ of a crown?—Ans. $\frac{3}{8}$.

102. What part of a yard is $\frac{1}{5}$ of an ell Eng.?—Ans. $\frac{1}{4}$.
103. What part of an ell Eng. is $\frac{1}{4}$ of a yard?—Ans. $\frac{4}{5}$.

ADDITION OF ARITHMETICAL FRACTIONS.

Rule.—Reduce compound and complex fractions to simple ones, and these fractions to a common denominator, over which place the sum of the numerators, which may be valued or reduced, as occasion requires.

Note. 1. When there are mixed numbers, the fractional part of them may be treated the same as the others; and, when the sum of all the fractions is found, the integral part may then be added.

2. To add fractions of different denominations, find their respective values by *Case* 12.

EXAMPLE I.

Add $\frac{1}{4}$, $\frac{7}{8}$, and $3\frac{5}{16}$, together.

Method 1.

$\frac{1}{4} + \frac{7}{8} + \frac{5}{16}$.

$3 \times 8 \times 16 = 384$*
$7 \times 4 \times 16 = 448$
$5 \times 4 \times 8 = 160$

$4 \times 8 \times 16 = \frac{992}{512} = 1\frac{4}{16}$

Then $1\frac{4}{16} + 3 = 4\frac{4}{16}$.—Ans.

Or, *Method* 2.

16 least com. mult. per *Case* 6.

Then $\frac{16 \times 3}{4} = 12$

$\frac{16 \times 7}{8} = 14$

And $\frac{16 \times 5}{16} = 5$

$\frac{31}{16} = 1\frac{4}{16}$

Then $1\frac{4}{16} + 3 = 4\frac{4}{16}$.—Ans.

* See Elucidation to Case 5.

EXAMPLE II.

What is the sum of $\frac{3}{8}$ of a pound and $\frac{1}{3}$ of a shilling?

		s.	d.	
£3	÷ 8 =	7	6	per *Note* 2
And 1*s.*	÷ 3 =	0	4	
		7	10—Ans.	

EXAMPLES.

104. Add $\frac{3}{5}$, $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{5}{6}$ together.—Ans. $3\frac{1}{10}$.

105. Add $\frac{7}{8}$, $\frac{5}{9}$, $\frac{6}{11}$, and $\frac{8}{13}$, together.—Ans. $2\frac{5051}{10296}$.

106. Add $\frac{3}{7}$, $\frac{4}{11}$, $\frac{6}{13}$, and $\frac{13}{15}$, together.—Ans. $2\frac{4021}{15015}$.

107. Add $11\frac{1}{4}$, $3\frac{1}{7}$, and 5, together.—Ans. $19\frac{11}{28}$.

108. Add $\frac{3}{7}$ of $\frac{1}{2}$, $\frac{2}{5}$ of $\frac{1}{4}$ of $\frac{3}{7}$, and $5\frac{1}{4}$, together.—Ans. $6\frac{1}{35}$.

109. Add $\frac{2\frac{2}{3}}{3\frac{1}{3}}$, $\frac{3}{\frac{1}{3}}$, and $\frac{\frac{2}{3}}{5\frac{1}{3}}$, together.—Ans. $4\frac{11}{47}$.

110. What is the sum of $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{4}{5}$, of a pound sterling?—Ans. £1 16*s.* 8*d.*

111. What is the sum of $\frac{3}{14}$, $\frac{1}{16}$, and $\frac{1}{44}$, of a shilling?—Ans. 2*s.* $1\frac{1}{14}$*d.*

112. What is the sum of $\frac{4}{100}$ of a ton, and $\frac{14}{112}$ of a cwt.?—Ans. 18 cwt. 1 lb. 13 oz. 12 dr.

113. What is the sum of $\frac{1}{4}$ of a mile, $\frac{1}{4}$ of a fur., $\frac{1}{10}$ of a pole, and $\frac{2}{11}$ of a yard?—Ans. 6 fur. 10 po. 2 yd. 3 in. $2\frac{6}{11}$ b. c.

114. What is the sum of $\frac{1}{73}$ of a common year and $\frac{1}{61}$ of a leap year?—Ans. 46 da.

SUBTRACTION OF ARITHMETICAL FRACTIONS.

Rule.—Reduce the fractions to a common denominator, as directed in Addition, over which place the difference of the numerators.

Note. 1. To subtract a proper fraction from unity, subtract the numerator from the denominator, under which difference place the denominator; thus, $1 - \frac{1}{3} = \frac{2}{3}$.

2. To subtract a proper fraction from any integer, subtract the numerator from the denominator, and the remainder placed over the denominator gives the fraction, which is to be annexed to the integer made less by unity; thus, $2 - \frac{4}{7} = 1\frac{3}{7}$.

3. In subtracting mixed numbers, if the lower numerator be less than the upper, their difference is the numerator of the remaining fraction; thus, $4\frac{5}{7} - 2\frac{4}{7} = 2\frac{1}{7}$.

4. But if the lower numerator be greater than the upper, the difference between the lower and the denominator added to the upper numerator, will be the numerator of the remaining fraction, for which 1 must be carried to the lower integer; thus, $3\frac{2}{5} - 1\frac{3}{5} = 1\frac{4}{5}$.

5. To subtract fractions of different denominations, find their respective values, as directed in *Case* 12, and afterwards proceed as in Compound Subtraction.

EXAMPLE I.

From $13\frac{3}{4}$ take $7\frac{1}{8}$.

First $\frac{3}{4} = \frac{6}{8}$, per *Case* 7.

Then $13\frac{6}{8} - 7\frac{1}{8} = 6\frac{5}{8}$.—Ans.

EXAMPLE II.

From $\frac{3}{5}$ of a pound, take $\frac{1}{3}$ of a shilling.

	s.	*d.*	
£$\frac{3}{5}$ =	7	6	per *Case* 12.
$\frac{1}{3}$*s.* =	0	4	
	7	2.—Ans.	

EXAMPLES.

115. From $\frac{5}{7}$ take $\frac{3}{7}$.—Ans. $\frac{2}{7}$.

116. From $\frac{3}{4}\frac{4}{3}$ take $\frac{1}{12}$.—Ans. $\frac{1}{156}$.

117. From 7 take $\frac{3}{4}$ of $\frac{4}{5}$.—Ans. $6\frac{17}{20}$.

118. From $13\frac{1}{2}$ take $\frac{1}{4}$ of $2\frac{1}{3}$.—Ans. $12\frac{23}{24}$.

119. From $\frac{3}{\frac{7}{9}}$ take $\frac{2\frac{1}{8}}{3\frac{1}{9}}$.—Ans. $3\frac{19}{124}$.

120. From $\frac{3}{4}$ of a pound, take $\frac{2}{3}\frac{3}{4}$ of a shilling.—Ans. 15*s.* $8\frac{1}{4}$*d.*

121. From $\frac{14}{15}\frac{3}{5}$ of a ton, take $\frac{14}{16}\frac{5}{14}$ of a cwt.—Ans. 17 cwt. 2 qr. 26 lb. 2 oz. 4 dr.

122. From $\frac{5}{7}\frac{3}{1}$ of a leap-year, take $\frac{3}{7}\frac{1}{2}$ of a common one.—Ans. 15 da.

MULTIPLICATION OF ARITHMETICAL FRACTIONS.

Rule.—Reduce integers and mixed numbers to improper fractions; and compound and complex fractions to simple ones; then place all the numerators, with the sign of multiplication between them, above a horizontal line, and all the denominators below it; then abbreviate, as in *Case* 4.

Note. 1. To multiply *a fraction by an integer*, either *divide* the denominator by the integer, or multiply the *numerator* by it; thus, multiply $\frac{5}{12}$ by 4, then $\frac{5}{12 \div 4} = \frac{5}{3} = 1\frac{2}{3}$; or $\frac{5 \times 4}{12} = \frac{20}{12} = \frac{5}{3} = 1\frac{2}{3}$.

2. To multiply *an integer by a mixed number*, multiply by the *integer* and *fraction* separately, and add the products; thus, multiply 5 by $3\frac{1}{4}$, then $3 \times 5 = 15$, and $5 \times \frac{1}{4} = 1\frac{1}{4}$, then $15 + 1\frac{1}{4} = 16\frac{1}{4}$; or, agreeably to the rule, $5 = \frac{5}{1}$, and $3\frac{1}{4} = \frac{13}{4}$, then $\frac{5}{1} \times \frac{13}{4} = \frac{65}{4} = 16\frac{1}{4}$.

3. If a *proper fraction be multiplied by a proper fraction*, the product will be less than either of the factors, as $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$; consequently, any number, either *whole* or *mixed*, multiplied by *a proper fraction*, will produce a product *less* than the multiplicand, as $3 \times \frac{1}{4} = \frac{3}{4}$, which is less than 3; and, if multiplied by *an improper fraction*, the product will be *more* than the multiplicand, as $3\frac{1}{2} \times \frac{4}{3} = \frac{7}{2} \times \frac{4}{3} = \frac{28}{6} = 4\frac{2}{3}$, which is more than $3\frac{1}{2}$.

EXAMPLE.

Multiply $\frac{3}{4}$ of $\frac{1}{2}$ by $7\frac{1}{2}$.

First, $\frac{3}{4}$ of $\frac{1}{2} = \frac{3}{8}$, per *Case* 4, and $7\frac{1}{2} = \frac{15}{2}$, per *Case* 1.

Then $\frac{1 \times 15}{2 \times 2} = \frac{15}{4} = 3\frac{3}{4}$—Ans.

Or,

$$\frac{\overline{2} \times \overline{3} \times 15}{\underline{8} \times 4 \times \underline{2}} = \frac{15}{4} = 3\frac{3}{4}.\text{—Ans.}$$

EXAMPLES.

123. Multiply $\frac{1}{2}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{5}{9}$, together.—Ans. $\frac{7}{18}$.

124. Multiply $\frac{5}{6}$ of $\frac{1}{2}$ of $\frac{3}{5}$, by $\frac{4}{5}$ of $\frac{1}{3}$ of $\frac{3}{4}$.—Ans. $\frac{1}{15}$.

125. Multiply $3\frac{1}{2}$ by $\frac{1}{7}$ of $4\frac{3}{4}$.—Ans. $2\frac{1}{2}$.

126. Multiply $5\frac{3}{4}$, $\frac{1}{3}$ of $10\frac{1}{2}$, and $\frac{3\frac{1}{2}}{2\frac{1}{4}}$, together.—Ans. $9\frac{17}{24}$.

127. Multiply $\frac{\frac{1}{2}}{3\frac{1}{2}}$ by $\frac{2}{3}$ of $\frac{3}{\frac{7}{8}}$.—Ans. $\frac{8}{21}$.

128. Multiply 700 by $\frac{1}{7}$ of 90.—Ans. 9000.

DIVISION OF ARITHMETICAL FRACTIONS.

Rule.—Reduce the fractions, &c. as before directed, and invert the divisor; then proceed precisely as in Multiplication.

Note. 1. When the numerator and denominator of the dividend are multiples of the numerator and denominator of the divisor respectively, to divide the numerator and denominator of the dividend by the numerator and denominator of the divisor respectively, is preferable to the general rule; thus, $\frac{12}{16} \div \frac{3}{4} = \frac{3}{4} = 1\frac{1}{4}$.

2. To divide *a fraction by an integer*, either divide the numerator, or multiply the denominator by the integer; thus, $\frac{4}{7} \div 2 = \frac{2}{7}$, or $\frac{4}{7 \times 2} = \frac{4}{14} = \frac{2}{7}$.

3. To divide *an integer by a fraction*, multiply by the denominator, and divide the product by the numerator thus, $2 \div \frac{4}{7} = \frac{2 \times 7}{4} = \frac{14}{4} = \frac{7}{2} = 3\frac{1}{2}$.

4. To divide *a proper fraction by a proper fraction*, the quotient will be *greater* than either divisor or dividend, as $\frac{2}{3} \div \frac{1}{3} = \frac{2}{1} = 2$, which is greater than either $\frac{2}{3}$ or $\frac{1}{3}$; consequently, if a whole or mixed number be divided by a proper fraction, the quotient will be *greater* than the dividend; and, if a proper fraction be divided by a whole or mixed number, the quotient will be *less* than the dividend.

EXAMPLE.

Divide $3\frac{1}{2}$ by $1\frac{3}{4}$.

First, $3\frac{1}{2} = \frac{7}{2}$, and $1\frac{3}{4} = \frac{7}{4}$.

Then, $\frac{7 \times 4}{2 \times 7} = 2$.—Ans.

EXAMPLES.

129. Divide $\frac{1}{7}$ by $\frac{1}{4}$.—Ans. $\frac{4}{7}$.

130. Divide $\frac{4}{5}$ by $\frac{1}{4}$.—Ans. $1\frac{7}{5}$.

131. Divide $\frac{2}{3}$ of $\frac{1}{3}$ of $\frac{4}{5}$ by $\frac{1}{4}$ of $\frac{8}{15}$ of $\frac{2}{3}$.—Ans. $1\frac{8}{15}$.

132. Divide $7\frac{1}{8}$ by $\frac{3}{5}$ of $4\frac{1}{4}$.—Ans. $1\frac{11}{100}$.

133. Divide $\frac{9}{91}$ by $\frac{\frac{1}{8}}{3\frac{1}{4}}$.—Ans. $2\frac{4}{7}$.

DECIMAL FRACTIONS.*

Definition.—A Decimal Fraction is such a one as may have 10, 100, 1000, &c. or 1 with as many ciphers annexed as there are places in the decimal, as its denominator, and which are expressed by writing the numerator only, with a period point on its left; thus, $\frac{5}{10} = .5$; $\frac{27}{100} = .27$; and $\frac{35}{1000} = .035$, &c.

Note. 1. A cypher *prefixed* to a decimal decreases its value in a ten-fold, and two, in a hundred-fold proportion, &c.; thus, $.5 = \frac{5}{10}$, but $.05 = \frac{5}{100}$, and $.005$, $= \frac{5}{1000}$, &c.; but to *annex* cyphers neither increases nor diminishes it; as $.5 = \frac{5}{10}$, and $.50 = \frac{50}{100} = \frac{5}{10}$, also.

2. From the above observations, it is evident that

* The celebrated John Muller, *alias* Regiomontanus, seems to have been the first who brought Decimal Fractions into use.

the value of decimals depends altogether upon the distance of the significant part of them, from the decimal point; and that the use of cyphers in decimals, as well as in integers, is to bring the significant figures to the places which their value indicates; it is also obvious that the first decimal place contains the tenths, the second the hundredths, and the third the thousandths, &c. of unity.

REDUCTION OF DECIMAL FRACTIONS.

Case 1.—To reduce Decimal Fractions to equivalent arithmetical ones.

Rule.—Take the decimal as the numerator, and 1, with as many cyphers annexed to it as there are places in the decimal for the denominator; this reduced to its lowest terms will be the fraction required.

EXAMPLE.

Reduce .125 to an equivalent arithmetical fraction.

$.125 = \frac{125}{1000} = \frac{25}{200} = \frac{5}{40} = \frac{1}{8}$.—Ans.

Remark.—The reason of this rule is evident from the definition and notation of a decimal fraction.

Case 2.—To reduce an arithmetical fraction to an equivalent decimal one.

Rule.—Annex cyphers to the numerator, and divide by the denominator.

Note.—If there be not as many figures in the quotient as the annexed cyphers, the defect must be supplied by prefixing cyphers.

EXAMPLE.

Reduce $\frac{1}{8}$ to an equivalent decimal fraction.

8) 1000

.125.—Ans.

Elucidation.—The numerator of an arithmetical fraction must always be understood to be divided by

the denominator, and this division is actually performed when it is reduced to a decimal fraction; thus, $\frac{1}{8}$ in the above example, or 1 divided by 8 is equal to 1000 thousandth parts divided by 8, which produces 125 thousandth parts.

Case 3.—To reduce lower denominations of money, weight, or measure, to decimals of higher denominations.

Rule.—Annex cyphers to the lower denomination, and divide by as many as make one of the higher, or

If there be several denominations given, begin at the lowest and bring it to the next higher: to this, prefix the given number of its name, and reduce it to the next higher, and so on till you produce the denomination wanted.

EXAMPLE I.

Reduce 3*d.* to the decimal of a shilling.

12) 300

.25.—Ans.

Remark.—As .25 ($= \frac{25}{100}$) $= \frac{1}{4}$, and as 3 pence are the fourth of a shilling, the result obtained must be correct.

EXAMPLE II.

Reduce 17*s.* 6*d.* to the decimal of a pound.

12) 60

20) 17.500

.875.—Ans.

Remark.—The same result would have been produced had we made the number of sixpences in 17*s.* 6*d.* the numerator, and those in a pound the denominator of an arithmetical fraction, and then reduced this to a decimal; thus, 17*s.* 6*d.* = 35 sixp. and 20*s.* = 40; then $\frac{35}{40} = \frac{7}{8} = .875$, as before, or 7 h. c. ÷ 8 h. c. $= \frac{7}{8} = .875$, as before; or,

Agreeably to a method given by Professor Lesslie,

in his "Philosophy of Arithmetic," 17*s.* 6*d.* multiplied continually by 10 till there be no shillings or pence left, will produce in pounds the figures of the decimal sought.

	£	*s.*	*d.*	
Thus,	0	17	6	
			10	
	8	15	0	
			10	
	87	10	0	
			10	
	875	0	0	as before.

2.—To reduce shillings, pence, and farthings, to the decimal of a pound, of three places, *mentally*.

Rule.—Write down half the number of shillings for the first decimal place, and the number of farthings in the remainder (increased by 1 if it amount to 24, by 2 if to 48, and by 3 if to 72,) gives the other two places when the shillings are even, and the number of thousandths to be added when the shillings are odd.

EXAMPLE.

Reduce 3*s.* 6½*d.* to the decimal of a pound.

2 is the greatest even number to 3 ∴ $\frac{3}{2} = 1$, the first place; and 1*s.* 6½*d.* = 74 far. and 74 + 3 = 77, for the second and third places; the decimal is therefore .177.

Case 4.—To find the value of a decimal.

Rule.—Multiply the given decimal by as many of the next lower name as make one of the given one, observing to mark off as many places as are in the given decimal; then multiply by as many of the next lower name as make one of this, and so on to the lowest necessary name; the several denominations on the left of the points, collectively, contain the answer.

EXAMPLE.

Find the value of .875 of a pound sterling.

$$\begin{array}{r} .875 \\ 20 \\ \hline 17.500 \\ 12 \\ \hline 6.000 \end{array}$$

The answer is therefore 17*s.* 6*d.*

2. To find the value of a decimal of a pound, *mentally*.

Rule.—As this is the converse of the second part of *Case* 3, double the 1st figure or place of 10ths for shillings; and, if the second place be 5, or more, reckon another shilling; then call the remaining figures in the second and third places so many farthings (abating 1 when they amount to 24, 2 when they are 48, and 3 when they are 72); the sum of these results will be the required value.

EXAMPLE.

Find the value of .177 of a pound.

First, double of 1 is 2 for the shillings;

But the second place is more than 5 ∴ we must reckon 3*s.*

And the remaining places $27 - 1 = 26$ far. $= 6\frac{1}{2}d.$

∴ $3s. + 6\frac{1}{2}d. = 3s. 6\frac{1}{2}d.$—Ans.

EXAMPLES TO CASE 1.

1. Produce arithmetical fractions for .14, .245, and .012.—Ans. $\frac{7}{50}$, $\frac{49}{200}$, and $\frac{3}{250}$.

2. Produce arithmetical fractions for .005, .0175, and .00025.—Ans. $\frac{1}{200}$, $\frac{7}{400}$, and $\frac{1}{4000}$.

3. Produce arithmetical fractions for .1015, .0105 and .7025.—Ans. $\frac{203}{2000}$, $\frac{21}{2000}$, and $\frac{281}{400}$.

EXAMPLES TO CASE 2.

4. What are the decimal expressions for $\frac{7}{50}$, $\frac{49}{200}$, and $\frac{3}{250}$.—Ans. .14, .245, and .012.

5. What are the decimal expressions for $\frac{1}{200}$, $\frac{7}{400}$, and $\frac{1}{4000}$.—Ans. .005, .0175, and .00025.

6. What are the decimal expressions for $\frac{101}{1000}$, $\frac{21}{1000}$, and $\frac{711}{1000}$.—Ans. .1015, .0105, and .7025.

EXAMPLES TO CASE 3.

7. Reduce $\frac{1}{4}$ of a penny to the decimal of a shilling. —Ans. .0625.

8. Reduce 7*s*. 6*d*. to the decimal of a pound.—Ans. .375.

9. Reduce 12*s*. 6*d*. to the decimal of a pound.—Ans. .625.

10. Reduce 13*s*. 9$\frac{1}{4}$*d*. to the decimal of a pound.—Ans. .690625.

11. Reduce 12 cwt. 3 qrs. to the decimal of a ton.—Ans. .6375.

12. Reduce 4 yds. 2 na. to the decimal of an ell French.—Ans. .75.

13. Reduce 6 h. 1 m. 48 sec. to the decimal of a day.—Ans. .25125.

EXAMPLES TO THE SECOND PART OF CASE 3.

Reduce, mentally, to the decimal of a pound,

14. 2*s*. 6*d*.—Ans. .125.
15. 14*s* 9*d*.—Ans. .737.
16. 17*s*. 6*d*.—Ans. .875.
17. 16*s*. 6*d*.—Ans. .825.
18. 8*s*. 7$\frac{1}{2}$*d*.—Ans. .432.
19. 11*s*. 7$\frac{1}{2}$*d*.—Ans. .581.
20. 4*s*. 9$\frac{1}{2}$*d*.—Ans. .239.
21. 7*s*. 4$\frac{1}{2}$*d*.—Ans. .368.
22. 9*s*. 6$\frac{1}{2}$*d*.—Ans. .476.

EXAMPLES TO CASE 4.

23. Find the value of .0625 of a shilling.—Ans. 3 farthings.

24. Find the value of .375 of a pound.—Ans. 7*s*. 6*d*.

25. Find the value of .625 of a pound.—Ans. 12*s*. 6*d*.

26. Find the value of .690625 of a pound.—Ans. 13*s*. 9$\frac{1}{4}$*d*.

27. Find the value of .6375 of a ton.—Ans. 12 cwt. 3 qrs.

28. Find the value of .75 of an ell French.—Ans. 4 qrs. 2 na.

29. Find the value of .25125 of a day.—Ans. 6 h. 1 m. 48 sec.

EXAMPLES TO THE SECOND PART OF CASE 4.

Find the values of the following decimals of a pound, *mentally*.

30. .125.—Ans. 2*s*. 6*d*.
31. .737.—Ans. 14*s*. 9*d*.
32. .875.—Ans. 17*s*. 6*d*.
33. .825.—Ans. 16*s*. 6*d*.
34. .432.—Ans. 8*s*. 7¾*d*.
35. .581.—Ans. 11*s*. 7½*d*.
36. .239.—Ans. 4*s*. 9½*d*.
37. .368.—Ans. 7*s*. 4½*d*.
38. .476.—Ans. 9*s*. 6¼*d*.

ADDITION OF DECIMAL FRACTIONS.

Rule.—Arrange the numbers under each other, so that the decimal points may fall under one another in a perpendicular line, and find their sums precisely as in integers.

EXAMPLE.

Add 2.148. 3.1, .005, and 173.01674, together.

2.148
3.1
.005
173.01674

178.26974.—Ans.

EXAMPLES.

What is the sum of the following numbers?

39. 16.4 + 13.09 + 1.004 + 17. + 5.036 + .17.—Ans. 52.7.

40. 31.8 + 174.56 + 1.812 + 16.008 + .0001.—Ans. 224.1801.

41. 1.74 + 16.32 + 123.4 + .0105 + 1.760.—Ans. 143.2305.

42. $2\frac{1}{2} + 1\frac{2}{5} + 4\frac{3}{5} + 12\frac{1}{4} + 3\frac{4}{5}$.—Ans. 24.325.

43. Add and value £13.174 and £471.31.—Ans. £484 9*s*. 8*d*.

44. Add and value £1.15 and £8.75, mentally.—Ans. £9 18*s*.

45. Reduce, mentally, and add and value £17 5*s*. 6*d*., £12 17*s*. 9*d*., and £30 14*s*. 5*d*.—Ans. £60 17*s*. 7½*d*.

46. What is the sum of a tenth and a ten thousandth?—Ans. .1001.

SUBTRACTION OF DECIMAL FRACTIONS.

Rule.—Place the less number under the greater, as directed in addition, and proceed with the subtraction as in simple numbers.

Note.—The pupil may annex cyphers to the right of the decimal of the greater number, when it does not contain as many places as the less.

EXAMPLE.

From 176.13, take 12.1654.

176.1300, per *Note.*
12.1654

163.9646.—Ans.

EXAMPLES.

47. Subtract 17.4 from 18.04.—Ans. .64.
48. Subtract .00012 from 1.—Ans. .99988.
49. Subtract .00001 from 10000.—Ans. 9999.99999.
50. Subtract $12\frac{1}{2}$ from $23\frac{1}{4}$.—Ans. 11.3.
51. Subtract £11 16*s.* $4\frac{1}{2}$*d.* from 20 guineas, decimally, and value the remainder.—Ans. £9 3*s.* $7\frac{1}{2}$*d.*
52. Subtract £12 17*s.* 9*d.* from £17 5*s.* 6*d.*, decimally, and value the remainder mentally.—Ans. £4 7*s.* 9*d.*
53. Subtract one-thousandth from one-tenth.—Ans. .099.

MULTIPLICATION OF DECIMAL FRACTIONS.

Rule.—Place the factors under each other, and multiply as in integers; point off from the right of the product as many places for a decimal as there are places in both factors; if the product have not so many places, prefix a cypher for each deficient figure.

Note.—To multiply by any product of 10*s.* we have but to remove the decimal point as many places towards the right as there are cyphers in the multiplier.

EXAMPLE I.

Multiply 1.5 by .25.

1.5
.25

75
30

.375.—Ans.

Elucidation.—The truth of the rule is evident; for, let us turn the factors into arithmetical fractions, and we have $1.5 = 1\frac{5}{10} = 1\frac{1}{2} = \frac{3}{2}$, and $.25 = \frac{25}{100} = \frac{1}{4}$, then $\frac{3}{2} \times \frac{1}{4} = \frac{3}{8} = .375$, as before.

EXAMPLE II.

Multiply .27 by .04.

.27
.04

.0108.—Ans.

Remark.—As each factor contains two decimal places, the product should therefore have four, and for that reason we have prefixed a cypher to the product.

EXAMPLES.

54. Multiply 12.3 by 3.21.—Ans. 39.483.
55. Multiply 7.107 by .005.—Ans. .035535.
56. Multiply 1.7 by .07.—Ans. .119.
57. Multiply 17.61 by 1.742.—Ans. 30.67662.
58. Multiply .154 by .0087.—Ans. .0013398.
59. Multiply £17 2*s.* 6*d.* by .0625.—Ans. £1 1*s.* 4¼*d.*
60. Multiply 18½ by 17½.—Ans. 313.95.

2. To limit the decimal places of the product to any proposed number.

Rule.—Set the unit's place of the multiplier under that place of the multiplicand which is to be retained in the product, and dispose of the other figures of the multiplier in the contrary order to that in which they would naturally stand.

Then multiply by the significant figures of the multiplier in their order, rejecting the figures to the right of each multiplying digit, and set down their products, so that their right-hand figures may fall perpendicularly under each other, observing to increase the first figure of each line by 1, if the product of the figure to the right of each multiplying digit amount to from 5 to 15; by 2, if from 15 to 25; and by 3, if from 25 to 35, &c.

EXAMPLE.

Multiply 3.141592 by 52.7438, so as to retain but 4 decimal places.

Common way.		*Contracted Method.*
3.141592		3.141592
52 7438		8347.25
25	132736	1570796
94	24776	62832
1256	6368	21991
21991	144	1256
62831	84	94
1570796	0	25
165.6995	001296.—Ans.	165.6994.—Ans.

Remark.—The contracted method produces very little more than $\frac{1}{10000}$ part of unity less than by the common way: but this is sufficiently near the truth for ordinary calculations; for what is $\frac{1}{10000}$ part even of a pound? not quite so much as $\frac{1}{10}$ of a farthing, which is the lowest of our current coin.

EXAMPLES.

61. Multiply 1.7421 by .0945, retaining 4 decimal places.—Ans. .1647.

62. Multiply 365,496 by 1.256, retaining 3 decimal places.—Ans. 459.062.

63. Multiply 8.741 by 1.0015, retaining 5 decimal places.—Ans. 8.75411.

64. Multiply 1.234 by 123.4, retaining 1 decimal place.—Ans. 152.3.

DIVISION OF DECIMAL FRACTIONS.

Rule.—Divide as in whole numbers, and cut off as many figures in the quotient as the decimal places in the dividend exceed those in the divisor.

If there be not figures enough in the quotient, the deficiency may be supplied by prefixing cyphers.

If there be a remainder, or if there be more decimal places in the divisor than in the dividend, cyphers may be annexed to the dividend, and the quotient carried as far as may be thought necessary.

Note. 1.—If the divisor and dividend have the same number of decimal places, the quotient will be a whole number.

2.—But, if the number of decimal places in each be not the same, they may be made so by annexing cyphers, and the quotient will also be a whole number.

3. In dividing by any multiple of 10, remove the decimal point on a place toward the left for each cypher in the divisor, which will be the quotient.

EXAMPLE I.

Divide 132.7656 by 1.1.

1.1) 132.7656

120.696—Ans.

EXAMPLE II.

Divide .375 by 1.5.

1.500) .37500 (0.25.—Ans.
3000

7500
7500

Elucidation.—The truth of this rule is evident, if we but find the quotient by arithmetical fractions; thus, $.375 = \frac{375}{1000} = \frac{375}{1000} = \frac{15}{40} = \frac{3}{8}$, and $1.5 = 1\frac{5}{10} = 1\frac{1}{10} = \frac{1}{2}$, then $\frac{3}{8} \div \frac{1}{2} = \frac{3}{4} = \frac{1}{4} = .25$, as before. See the converse of this in Multiplication.

EXAMPLES.

65. Divide 39.483 by 3.21.—Ans. 12.3.
66. Divide .035535 by 7.107.—Ans. .005.
67. Divide .119 by .07.—Ans. 1.7.
68. Divide 30.67662 by 17.61.—Ans. 1.742.
69. Divide .0013398 by .0087.—Ans. .154.
70. Divide 313.95 by 18½.—Ans. 17.25.

2. To limit the decimal places of the quotient so as to retain any proposed number.

Rule.—Take as many of the left-hand places of the divisor as will be equal to the number of integers and decimals in the quotient. Should there not be a sufficient number of places in the divisor, supply the deficiency by annexing cyphers: the remainders, divided by this divisor, made less by one place from the right at each step, till all the figures in the divisor be exhausted, will produce the required quotient. The carriage of the figure omitted must be added to the several products, as in Multiplication.

EXAMPLE.

Divide 6974.32 by 24.798, retaining 3 decimal places in the quotient.

Common way.

24.798) 6974.32 (281.245.—Ans.
49596
201472
198384
30880
24798
60820
49596
112240
99192
130480
123990
6490

Contracted Method.

```
24.7980 ) 6974.32 ( 281.245.—Ans.
          495960
          ------
          201472
          198384
          ------
            3088
            2480
            ----
             608
             496
             ---
             112
              99
              --
              13
              12
              --
               1
               -
```

EXAMPLES.

71. Divide .1647 by .0945, retaining 4 decimal places. —Ans. 1.7420.

72. Divide 459.062 by 1.256, retaining 3 decimal places.—Ans. 365.496.

73. Divide 8.75411 by 1.0015, retaining 2 decimal places.—Ans. 8.75.

74. Divide 152.3 by 1.234, retaining 1 decimal place.—Ans. 123.4.

PRACTICE.

Definition.—Practice is a short and expeditious method of calculating the value of goods by taking aliquot parts.

Note. 1.—*An aliquot part* of any number is that which being taken a certain number of times will exactly make that number; the number may, therefore, not unaptly, be called the multiple of any of its aliquot parts; thus, 5*s.* being contained in 20*s.* just 4 times, are, therefore, an aliquot part of its multiple, 20*s.*

2. *An aliquant part* is such a part of a number, which, however frequently repeated, will never make up the number exactly; thus, 3 is an aliquant part of 10, 3 threes being less than 10, and 4 threes more than 10.

3. The number of times that any aliquot part is contained in its multiple, is the *divisor* for finding the value of the same; thus, 5*s.* are contained just 4 times in a pound; therefore, any number of articles at 5*s.* each, divided by 4, will be the value of the same. Upon this all the rules of Practice are built.

4. The result is always of the same denomination of which the aliquot parts are taken, as, when we take parts of a pound, the answer is in pounds, shillings, pence, and farthings; and, if of a penny, the answer is in pence and farthings, &c.

TABLE I.—OF ALIQUOT PARTS.

OF A POUND.			OF A SHILLING.		OF A CWT.	
s.	*d.*				2 qrs.	are $\frac{1}{2}$
10	0	are $\frac{1}{2}$	6*d.*	are $\frac{1}{2}$	1	$\frac{1}{4}$
6	8	.... $\frac{1}{3}$	4	.... $\frac{1}{3}$	16 lbs. ..	$\frac{1}{7}$
5	0	.... $\frac{1}{4}$	3	.... $\frac{1}{4}$	14	$\frac{1}{8}$
4	0	.... $\frac{1}{5}$	2	.... $\frac{1}{6}$	8	$\frac{1}{14}$
3	4	.... $\frac{1}{6}$	$1\frac{1}{2}$	.... $\frac{1}{8}$	7	$\frac{1}{16}$
2	6	.... $\frac{1}{8}$	1	is $\frac{1}{12}$	OF A QR.	
1	8	.... $\frac{1}{12}$	OF A TON.		14 lb.	are $\frac{1}{2}$
1	4	.... $\frac{1}{15}$	10 cwt.	are $\frac{1}{2}$	7	$\frac{1}{4}$
1	3	.... $\frac{1}{16}$	$6\frac{2}{3}$	$\frac{1}{3}$	4	$\frac{1}{7}$
1	0	is $\frac{1}{20}$	5	$\frac{1}{4}$	$3\frac{1}{2}$	$\frac{1}{8}$
0	10	are $\frac{1}{24}$	4	$\frac{1}{5}$	2	$\frac{1}{14}$
0	8	.... $\frac{1}{30}$	$3\frac{1}{3}$	$\frac{1}{6}$	OF A lb.	
0	6	.... $\frac{1}{40}$	$2\frac{1}{2}$	$\frac{1}{8}$	8 oz.	are $\frac{1}{2}$
0	4	.... $\frac{1}{60}$	$1\frac{1}{4}$	$\frac{1}{12}$	4	$\frac{1}{4}$
0	3	.... $\frac{1}{80}$	1	is $\frac{1}{20}$	2	$\frac{1}{8}$
0	2	.... $\frac{1}{120}$				

TABLE II.—OF ALIQUOT PARTS.

OF A POUND.			OF A POUND.		OF A SHILLING.	
s.	*d.*		*s.*	*d.*	9*d.*	are $\frac{3}{4}$
18	4	are $\frac{11}{12}$	4	6 are $\frac{3}{40}$	8	$\frac{2}{3}$
17	6	..... $\frac{17}{18}$	3	8 ..... $\frac{1}{60}$	$7\frac{1}{2}$	$\frac{5}{8}$
16	8	..... $\frac{5}{6}$	3	6 ..... $\frac{9}{40}$	$4\frac{1}{2}$	$\frac{3}{8}$
16	0	..... $\frac{4}{5}$	2	9 ... .. $\frac{11}{60}$	OF A POUND, DECIMALLY.	
15	0	..... $\frac{3}{4}$	2	8 ..... $\frac{2}{60}$	10*s.* are	.5
13	4	..... $\frac{2}{3}$	2	4 ..... $\frac{7}{60}$	12	.6
12	6	..... $\frac{5}{8}$	2	3 ..... $\frac{1}{20}$	14	.7
12	0	..... $\frac{3}{5}$	1	10 .... $\frac{11}{120}$	16	.8
11	8	..... $\frac{7}{12}$	1	9 ..... $\frac{1}{30}$	18	.9
8	4	..... $\frac{5}{12}$	1	6 ..... $\frac{3}{40}$	22	.11
8	0	.... $\frac{2}{5}$	1	4 ..... $\frac{1}{30}$	24	.12
7	6	..... $\frac{3}{8}$	1	3 ..... $\frac{1}{40}$		
7	4	..... $\frac{11}{30}$	1	2 .... $\frac{7}{120}$		
5	6	..... $\frac{11}{40}$	OF A SHILLING.			
5	4	..... $\frac{2}{30}$	10$\frac{1}{2}$*d.*	are $\frac{7}{8}$		
4	8	.... $\frac{7}{30}$	10	$\frac{5}{6}$		

Multiply by the decimals; the result will be pounds.

General Rules for Commercial Calculations.

Case. 1.—When the price is an aliquot part.

Rule.—Divide the given quantity by the number of times the price is contained in a pound or penny,* and the result will either be pounds or pence.

EXAMPLES.

What is the value of 5496 yards of cloth, at 5*s.*, 3*d.*, and ¼.

1st, 5*s.* = ¼) 5496, at 5*s.*

£1374—Ans. *Note* 4.

2d. 3*d.* = $\frac{1}{80}$) 549,6

£68 14*s.*—Ans.

3d. ¼ = ¼) 5496, at ¼.

12) 1374

2,0) 11,4 6

£5 14 6.—Ans.

Case 2.—When the price is an aliquant part.

Rule.—Multiply the given quantity by the highest denomination of the price; and take parts for the lower ones; or,

Divide the given quantity by the number of times the nearest aliquot part to the price is contained in a pound or penny; and for the remainder of the price take other parts, either of a pound or penny,* or of the value which you have just found, and their sum will be the answer in pounds, shillings, or pence.

* Since 2*d.* = $\frac{1}{120}$, 3*d.* = $\frac{1}{80}$, 4*d* = $\frac{1}{60}$, 6*d.* = $\frac{1}{40}$, and 8*d.* = $\frac{1}{30}$ of a pound, to take the aliquot parts of a shilling, is altogether unnecessary, except where we first multiply by the shillings, and take parts for the rest.

EXAMPLES.

What is the value of 5496 yards, at 19*s.* 6*d.*, 4½*d.*, and ¾*d.* each.

Method 1.

1st. ½) 5496
19½

104424
2748

2,0) 10717,2

£5358 12*s.*—Ans.

Or, *Method* 2.

10*s.* = ½ | 5496, at 19*s.* 6*d.*
5*s.* = ¼ | 2748
4*s.* = ⅛ | 1374
6*d.* = ⅛ | 1090 4
137 8

£5358 12*s.*—Ans.

Or, *Method* 3.

2*s.* = $\frac{1}{10}$) 5496
7

8) 38472

4809 = 17*s.* 6*d.* *Tab.* 2.
549 12

£5358 12*s.*—Ans.

Or, *Method* 4.

Off 6*d.* = $\frac{1}{40}$) 5496
137 8

£5358 12.—Ans.

2d. 4d. = ½) 5496, at 4½d.

½ = ½) 1832
229

2,0) 206,1

£103 1.—Ans.

Or, 5496
3

8) 16488

2,0) 206,1

£103 1, *Tab.* 2.

Or, ⅔ = ½) 5496

½ = ½) 1374
687

2,0) 206,1

£103 1.—Ans.

3d. ½ = { ½ | 5496, at ¼d.
¼ = { ½ | 2748
1374

12) 4122

2,0) 34,3 6

£17 3 6.—Ans.

Or, ½ = ½) 5496

¼ = ½) 2748
1374

12) 4122

2,0) 34,3 6

£17 3 6—Ans.

Note. 5.—When the highest denomination in the price is unity, consider the given quantity of the same name as the highest given denomination; to which add the value of the remaining part of the price; thus,

5496, at £1 6*s.* 8*d.*, 1*s.* 6*d.*, and 1¼*d.*

1st. 6*s*. 8*d*. = ⅓) 5496, at £1 6*s*. 8*d*.
1832

£7328—Ans.

2d. 6*d*. = ½) 5496, at 1*s*. 6*d*.
2748

2,0) 824,4

£412 4.—Ans.

3d. ¼ = ¼) 5496, at 1¼*d*.
1374

12) 6870

2,0) 57,2 6

£28 12 6.—Ans.

Case 3.—1. When the price is an even number of shillings.

Rule.—Multiply the quantity by half the price, and double the first figure of the product for shillings.

2. When the price is an odd number of shillings.

Rule.—Find the value of the quantity at the nearest even number of shillings to those which are given, and add $\frac{1}{20}$ of the top line for the remaining shilling.

EXAMPLES.

What is the value of 5496, at 8*s*. and 9*s* ?

Method 1.

1st. 5496, at 8*s*.
.4

£2198 8—Ans. *Tab*. 2.

Or, *Method* 2.

4 = ⅕) 5496, *Case* 2.

1099 4
1099 4

£2198 8.—Ans.

Method 1.

2d. 1*s.* = $\frac{1}{20}$) 5496, at 9*s.*
4

2198 8
274 16

£2473 4.—Ans.

Or, *Method* 2.

5 = $\frac{1}{4}$ | 5496, *Case* 2.
4 = $\frac{1}{5}$ | 1374
1099 4

£2473 4.—Ans.

Or, *Method* 3.

10*s.* = $\frac{1}{2}$) 5496

Off 1*s.* = $\frac{1}{20}$) 2748
274 16

£2473 4.—Ans.

Note 6.—When the price, whether even or odd, is an aliquot part of a pound, such as 2*s.*, 4*s.*, 5*s.*, or 10*s.* it is preferable to treat it as such.

7. When the price is pounds only, the value is found by Simple Multiplication.

Case 4.—When there is a fraction in the given quantity.

Rule.—Calculate the value of the whole number as before directed, and add a proportional part of the price for the value of the fraction; or consider the given quantity as so many pounds, with the shillings and pence, &c. which the fraction is of a pound, and calculate as before: this is, perhaps, the better way.

Calculate 5496¼ yards, at 13*s.* 4*d.*

Method 1.

10*s.* = ½) 5496, at 13*s.* 4*d.*

3*s.* 4*d.* = ⅓) 2748
916
¼ of 13*s.* 4*d.* = 0 10

£3664 10.—Ans.

Or, *Method* 2.

6*s.* 8*d.* = ⅓) 5496 15

1832 5
1832 5

£3664 10.—Ans.

Or, *Method* 3.

Off 6*s.* 8*d.* = ⅓) 5496.75
1832.25

£3664.5.—Ans.

Case 5.—When the quantity is a compound number.

Rule.—To the product of the price by the highest denomination, add the parts of the price for the inferior denominations.

EXAMPLE.

Calculate 32 cwt. 3 qrs. 14 lb, at £3 16*s.* 8*d.* per cwt.

2 qrs. = ½	£3	16 8
		8
	30	13 4
		4
	122	13 4
1 qr. = ¼	1	18 4
14 lb. = ½	0	19 2
	0	9 7
	£126	0 5—Ans.

Or, *Method* 2.

Off 3*s.* 4*d.* = ¼) 32
4

128
5 6 8

2 qrs.= ½	122	13 4
1 qr. = ¼	1	18 4
14 lb. = ½	0	19 2
	0	9 7
	£126	0 5.—Ans.

Or, *Method* 3.

	cwt.	qr.	lb.		
	32	3	14		
		5	2½		
On 3*s.* 4*d.* = ¼	32	17	6	=	val. at £1.
			4		
	131	10	0	=	val. at £4 0 0
	5	9	7	=	val. at 0 3 4
	£126	0	5	=	val. at £3 16 8

Note. 8.—The third method is done by considering the number of the highest denomination as pounds, and the lower denominations are converted into shillings and pence by multiplying the qrs. by the value of 1 qr. which is ¼ of £1, or 5*s.*; and the lbs. by the value of 1 lb. which is $\frac{1}{112}$ of £1, or $\frac{1}{112}$ of 5*s.* = $\frac{60}{112}$ = $\frac{5}{9}$ = 2$\frac{1}{9}$.

EXAMPLES TO CASE 1.

1. 1234 lbs. at 10*s.*, 5*s.*, 2*s.*, 6*d.*, 3*d.*, ½*d*, and ¼*d.*, —Ans. 617*l.*, 308*l.* 10*s.*, 123*l.* 8*s.*, 30*l.* 17*s.*, 15*l.* 8*s.* 6*d.*, 2*l* 11*s.* 5*d.*, and 1*l.* 5*s.* 8½*d.*

2. 2345, at 6*s.* 8*d.*, 3*s* 4*d.*, 1*s.* 8*d.*, 4*d.*, 2*d.*, 1½*d.*, and ¼*d.*—Ans. 781*l.* 13*s.* 4*d.*, 890*l.* 16*s.* 8*d.*, 195*l.* 8*s.* 4*d*, 39*l.* 1*s.* 8*d.*, 19*l.* 10*s.* 10*d.*, 14*l.* 13*s.* 1½*d.*, and 2*l.* 6*s.* 10½*d.*

3. 3456, at 2*s.* 6*d.*, 4*s.*, 1*s.* 4*d.*, 1½*d.*, and ¼.—Ans. 432*l.*, 691*l.* 4*s.*, 230*l.* 8*s.*, 21*l.* 12*s.*, and 7*l.* 4*s.*

EXAMPLES TO CASE 2.

4. 6789 lbs., at 10*s.* 6*d.*, 5*s.* 10*d.*, 2*s.* 4*d*, 7*d.*, 5*d*, and ¾*d.*—Ans. 3564*l.* 4*s.* 6*d.*, 1980*l.* 2*s.* 6*d.*, 792*l.* 1*s.*, 198*l.* 0*s.* 3*d.*, 141*l.* 8*s.* 9*d.*, and 21*l.* 4*s.* 3¾*d.*

5. 7890 lbs., at 6*s.* 10*d.*, 7*s.* 9*d.*, 8*s.* 2*d.*, 9*d.*, 8*d.*, and ¾*d.*—Ans. 2695*l.* 15*s.*, 3057*l.* 7*s.* 6*d.*, 3221*l.* 15*s.*, 295*l.* 12*s.* 6*d.*, 263*l.*, and 24*l.* 13*s.* 1½*d.*

6. 8901 lbs., at 13*s.* 4*d.*, 16*s.* 8*d.*, 12*s.* 6*d.*, 17*s.* 6*d*, 19*s.* 6*d.*, and 2½*d.*—Ans. 5934*l.*, 7417*l.* 10*s.*, 5563*l.* 2*s.* 6*d.* 7788*l.* 7*s.* 6*d.*, 8678*l.* 9*s.* 6*d.*, and 83*l.* 8*s.* 11¼*d.*

7. 9012 lbs., at 2½*d.*, 3¾*d.*, 4½*d.*, 6¾*d.*, 7¾*d.*, and 9½*d.*—Ans. 93*l.* 7*s.* 6*d.*, 140*l.* 16*s.* 3*d.*, 168*l.* 19*s.* 6*d*, 253*l.* 9*s.* 3*d.*, 292*l.* 0*s.* 3*d.*, and 347*l.* 6*s.* 9*d.*

8. 1234 lbs., at 13*s.* 8*d.*, 18*s.* 6*d.*, 2*s.* 4½*d*, 3*s.* 11¼*d.*, and 2*s.* 11½*d.*—Ans. 843*l.* 4*s.* 8*d.*, 1141*l.* 9*s.*, 146*l.* 6*s.* 3*d.*, 242*l.* 18*s.* 10½*d.*, and 183*l.* 16*s.* 3½*d.*

9. 4567 cwts., at 2*l.* 6*s.* 8*d.*, 3*l.* 13*s.* 4*d.*, and 7*l.* 19*s.* 9*d.*—Ans. 10656*l.* 6*s.* 8*d.*, 16745*l.* 13*s.* 4*d.*, and 36478*l.* 18*s.* 3*d.*

10. 5678 cwt., at 17*l.* 11*s.* 5½*d.*, 13*l.* 14*s.* 3¾*d.*, and 15*l.* 19*s.* 10*d.*—Ans. 99779*l.* 0*s.* 5*d.*, 77877*l.* 6*s.* 4½*d.*, and 92239*l.* 18*s.* 8*d.*

11. 6789 cwt., at 12*l.* 18*s.* 11½*d.*, and 10*l.* 14*s.* 9¾*d.* —Ans. 87903*l.* 8*s.* 1½*d.*, and 72918*l.* 2*s.* 0¾*d.*

12. 7890 cwt., at 5*s.* 4½*d.*, 7*s.* 9¾*d.*, and 12*s.* 8½*d.* —Ans. 2120*l.* 8*s.* 9*d.*, 3082*l.* 0*s.* 7½*d.*, and 5013*l.* 8*s.* 9*d*

13. 8901 cwt., at 13*s.* 11¾*d.*, 17*s.* 2½*d.*, and 16*s.* 9¾*d.* — Ans. 6221*l.* 8*s.* 6¾*d.*, 7658*l.* 11*s.* 4½*d.*, and 7482*l.* 4*s.* 0¾*d.*

14. 1765 cwt., at 7½*d.*, 10½*d.*, 9½*d.*, and 11¾*d.* —Ans. 55*l.* 3*s.* 1½*d.*, 75*l.* 7*s.* 7½*d.*, 69*l.* 10*s.* 7½*d.*, and 86*l.* 14*s.* 10¾*d.*

15. 3297 cwt., at 1*l.* 18*s.* 9½*d.*, and 1*l.* 11*s.* 8¾*d.*— Ans. 6395*l.* 2*s.* 1½*d.*, and 5230*l.* 4*s.* 1¾*d.*

16. 7965 cwt., at 11*s.* 6¾*d.*, 7*s.* 1¾*d.*, and 1*s.* 1½*d.* — Ans. 4604*l.* 15*s.* 3¾*d*, 2845*l.* 16*s.* 6¾*d.*, and 418*l.* 0*s.* 7½*d.*

17. 5678 yards, at 10½*d.*, 7½*d.*, 11¾*d.*, and 8½*d.*— Ans. 248*l.* 8*s.* 3*d.*, 171*l.* 10*s.* 5½*d.*, 277*l.* 13*s.* 8½*d.*, and 200*l.* 19*s.* 5*d.*

18. 6789 yards, at 13*s.* 8*d* , 19*s.* 2*d.*, 16*s.* 4*d.*, and 12*s.* 8*d.*—Ans. 4639*l.* 3*s.*, 6506*l.* 2*s.* 6*d.*, 5544*l.* 7*s.*, and 4299*l.* 14*s.*

19. 7890 yards, at 1*s.* 2½*d.*, 2*s.* 3½*d.*, 3*s.* 4¾*d.*, and 4*s.* 8*d.*—Ans. 468*l.* 9*s.* 4½*d.*, 904*l.* 1*s.* 3*d.*, 1339*l.* 13*s.* 1½*d.* and 1841*l.*

20. 8901, at 11*s.* 4½*d.*, 13*s.* 8½*d.*, 17*s.* 9¾*d.*, and 19*s.* 11¾*d.*—Ans. 5062*l.* 8*s.* 10½*d.*, 6100*l.* 17*s.* 10½*d.*, 7927*l.* 9*s.* 0¾*d.*, and 8891*l.* 14*s.* 6¾*d.*

EXAMPLES TO CASE 3.

21. 9012 yards, at 2*s.*, 4*s.*, 6*s.*, 8*s.*, 10*s.*, and 12*s.* —Ans. 901*l.* 4*s.*, 1802*l.* 8*s.*, 2703*l.* 12*s.*, 3604*l.* 16*s.*, 4506*l.*, and 5407*l.* 4*s.*

22. 1234 yards, at 3*s.*, 5*s.*, 7*s.*, 9*s.*, 11*s.*, and 13*s.* — Ans. 185*l.* 2*s.*, 308*l.* 10*s.*, 431*l.* 18*s.*, 555*l.* 6*s.*, 678*l.* 14*s.*, and 802*l.* 2*s.*

23. 2345 yards, at 14*s.*, 16*s.*, 18*s.*, 20*s.*, 22*s.*, and 24*s.*—Ans. 1641*l.* 10*s.*, 1876*l.*, 2110*l.* 10*s.*, 2345*l.*, 2579*l.* 10*s.*, and 2814*l.*

24. 3456 yards, at 15*s.*, 17*s.*, 19*s.*, 21*s.*, 23*s.*, and 25*s.*—Ans. 2592*l.*, 2937*l.* 12*s.*, 3283*l.* 4*s.*, 3628*l.* 16*s.*, 3974*l.* 8*s.*, and 4320*l.*

EXAMPLES TO CASE 4.

25. 1745$\frac{1}{8}$ yards, at 1*s.* 8*d.*, 2*s.* 6*d.*, 3*s.* 4*d.*, 5*s.* 6*d.*, and 3*s.* 8*d.*—Ans. 145*l.* 9*s.* 2*d.*, 218*l.* 3*s.* 7$\frac{1}{2}$*d.*, 290*l.* 18*s.* 4*d.*, 480*l.* 0*s.* 3*d.*, and 320*l.* 0*s.* 2*d.*

26. 1396$\frac{1}{4}$ yards, at 17*s.* 6*d.*, 28*s.* 9*d.*, 33*s.* 4*d.*, and 11*s.* 2$\frac{1}{2}$*d.*—Ans. 1221*l.* 14*s.* 4$\frac{1}{2}$*d.*, 2007*l.* 2*s.* 2$\frac{1}{4}$*d.*, 2307*l.* 1*s.* 8*d.*, and 782*l.* 9*s.* 5$\frac{1}{16}$*d.*

27. 7369$\frac{3}{4}$ yards, at 1$\frac{1}{2}$*d.*, 3$\frac{1}{2}$*d.*, 4$\frac{1}{2}$*d.*, and 6$\frac{3}{4}$*d.*—Ans. 46*l.* 1*s.* 2$\frac{3}{8}$*d.*, 115*l.* 3*s.* 0$\frac{9}{16}$*d.*, 138*l.* 3*s.* 7$\frac{5}{8}$*d.*, and 207*l.* 5*s.* 5$\frac{1}{16}$*d.*

28. 8453$\frac{7}{8}$ yards, at 1*s.* 2$\frac{1}{8}$*d.*, 2*s.* 9$\frac{1}{4}$*d.*, 3*s.* 4$\frac{1}{2}$*d.*, and 1*s.* 4$\frac{1}{2}$*d.*—Ans. 510*l.* 15*s.* 1$\frac{3}{16}$*d.*, 325*l.* 16*s.* 6$\frac{1}{32}$*d.*, 1426*l.* 11*s.* 9$\frac{1}{16}$*d.*, and 581*l.* 4*s.* 0$\frac{9}{32}$*d.*

29. 17864$\frac{5}{8}$ yards, at 6*s.* 4$\frac{1}{2}$*d.*, 5*s.* 10$\frac{1}{2}$*d.*, 7*s.* 11$\frac{1}{2}$*d.*, and 1*s.* 2$\frac{1}{2}$*d.*—Ans. 571*l.* 8*s.* 0$\frac{9}{10}$*d.*, 523*l.* 0*s.* 2$\frac{7}{10}$*d.*, 707*l.* 5*s.* 6$\frac{1}{20}$*d.*, and 109*l.* 19*s.* 3$\frac{1}{10}$*d.*

30. 7325$\frac{7}{12}$ yards, at 12*s.* 2$\frac{1}{2}$*d.*, 7*l.* 10*s.* 3$\frac{1}{4}$, and 5*l.* 18*s.* 11*d.*—Ans. 4471*l.* 13*s.* 1$\frac{2}{24}$*d.*, 55041*l.* 1*s.* 6$\frac{7}{48}$*d.*, and 6928*l.* 15*s.* 7$\frac{5}{12}$*d.*

EXAMPLES TO CASE 5.

31. 176 cwt. 3 qrs. 14 lbs., at 7*l.* 6*s.* 8*d.*—Ans. 1297*l.* 1*s.* 8*d.*

32. 132 cwt. 1 qr. 7 lbs., at 11*l.* 17*s.* 6*d.*—Ans. 1571*l.* 4*s.* 2$\frac{3}{8}$*d.*

33. 144 cwt. 2 qrs. 1 lb., at 10*l.* 13*s.* 2$\frac{1}{2}$*d.*—Ans. 1540*l.* 10*s.* 6$\frac{3}{32}$*d.*

34. 156 cwt. 3 qrs. 24 lbs., at 11*l.* 10*s.* 9*d.*—Ans. 1810*l.* 19*s.* 6$\frac{3}{28}$*d.*

35. 112 cwt. 0 qrs. 18 lbs., at 9*l.* 16*s.* 7$\frac{1}{4}$*d.*—Ans. 1102*l.* 15*s.* 11$\frac{55}{224}$*d.*

36. 70 oz. 5 dwt. 6 gr., at 5*s.* per oz.*—Ans. 17*l.* 11*s.* 3¾*d.*

37. 729 oz. 11 dwt. 17 gr., at 4*s.* 9*d.* per oz.—Ans. 173*l.* 5*s.* $6\frac{7}{24}$*d.*

38. 357 oz. 19 dwt. 23 gr., at 5*s.* 1¼*d.* per oz.—Ans. 91*l.* 7*s.* $2\frac{11}{15}\frac{17}{24}$*d.*

39. 134 lb. 14 dwt. 1 gr., at 3*s.* 9½*d.* per oz.—Ans. 325*l.* 14*s.* $5\frac{8}{6}\frac{7}{40}$*d.*

40. 965 oz. 16½ gr., at 4*s.* 4½*d.* per oz.—Ans. 211*l.* 2*s.* $0\frac{33}{152}$*d.*

QUESTIONS IN PRACTICE, PROMISCUOUSLY ARRANGED.

41. Calculate 7429 lbs., at ¾*d.*, 9*d.*, 15*s.*, and 17*l.*—Ans. 23*l.* 4*s.* 3¾*d.*, 278*l.* 11*s.* 9*d.*, 5571*l.* 15*s.*, and 126293*l.*

42. Calculate 2768 lbs., at 2*d.*, 7*d.*, 11*d.*, and 10*d.*—Ans. 23*l.* 1*s.* 4*d.*, 80*l.* 14*s.* 8*d.*, 126*l.* 17*s.* 4*d.*, and 115*l.* 6*s.* 8*d.*

43. Calculate 1249 lbs., at 18*s.*, 21*s.*, 23*s.*, 24*s.*, and 25*s.*—Ans. 1124*l.* 2*s.*, 1311*l.* 9*s.*, 1436*l.* 7*s.*, 1498*l.* 16*s.*, and 1561*l.* 5*s.*

44. Calculate 7968 lbs., at 17*s.* 4½*d.*, 11*s.* 9½*d.*, 16*s.* 4½*d.*, and 18*s.* 2½*d.*—Ans. 6922*l.* 4*s.*, 4689*l.* 10*s.*, 6523*l.* 16*s.*, and 7262*l.* 10*s.*

45. Calculate 6954 lbs., at 137*l.* 15*s.* 8½*d.* and 765*l.* 18*s.* 10½*d.* — Ans. 958159*l.* 15*s.* 9*d.*, and 5319526*l.* 1*s.* 7½*d.*

46. Calculate $1897\frac{1}{8}$ lbs., at 5*s.* 6*d.*, 3*s.* 9*d.*, 4*s.* 11*d.*, and 7*s.* 6½*d.*—Ans. 534*l.* 9*s.* 2½*d.*, 355*l.* 14*s.* 2½*d.* 466*l.* 7*s.* $6\frac{3}{8}$*d.*, and 717*l.* 7*s.* $0\frac{3}{32}$*d.*

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* In calculating sums in Troy Weight, reckon 1*s.* for every dwt., and ½*d.* for every grain, and afterwards take the aliquot parts for the price; thus, 70 oz. 5 dwt. 6 gr. = 70*l.* 5*s.* 3*d.* and 5*s.* = ¼) 70 5 3

£17 11 3¾.—Ans.

47. Calculate 3756$\frac{7}{12}$ lbs., at 4*s.* 2*d.*, 7*s.* 9*d.*, 8*s.* 6*d.*, and 9*s.* 11*d.* — Ans. 782*l.* 12*s.* 8$\frac{1}{2}$*d.*, 1455*l.* 13*s.* 6$\frac{1}{2}$*d.*, 1596*l.* 10*s.* 11$\frac{1}{2}$*d.* and 1862*l.* 11*s.* 9$\frac{11}{12}$*d.*

48. Calculate 4963$\frac{1}{2}$ lbs., at 1*l.* 14*s.* 2*d.* and 1*l.* 17*s.* 9$\frac{1}{2}$*d.*—Ans. 8479*l.* 9*s.* 8*d.* and 9379*l.* 2*s.* 8$\frac{1}{2}$*d.*

49. Calculate 1289$\frac{5}{14}$ lbs., at 13*s.* 8$\frac{1}{2}$*d.*, 14*s.* 8*d.*, and 19*s.* 10*d.*—Ans. 883*l.* 14*s.* 3$\frac{3}{7}$*d.*, 945*l.* 9*s.* 11*d.*, and 1278*l.* 11*s.* 4$\frac{4}{7}$*d.*

50. Calculate 7432$\frac{4}{15}$ lbs., at 17*s.* 4$\frac{1}{2}$*d.*, 19*s.*, and 24*s.*—Ans. 6457*l.* 0*s.* 3$\frac{4}{5}$*d.*, 7060*l.* 18*s.* 1$\frac{1}{5}$*d.*, and 8919*l.* 0*s.* 9$\frac{3}{5}$*d.*

51. Calculate 17 cwt. 1 qr. 19 lb., at 5*l.* 17*s.* 6*d.* per cwt.—Ans. 102*l.* 6*s.* 9$\frac{13}{56}$*d.*

52. Calculate 32 cwt. 2 qr. 18 lb., at 3*l.* 12*s.* 8*d.* per cwt.—Ans. 118*l.* 13*s.* 4$\frac{1}{7}$*d.*

53. Calculate 98 cwt. 3 qr. 14 lb., at 1*l.* 17*s.* 4$\frac{1}{2}$ *l.* per cwt.—Ans. 184*l.* 11*s.* 4*d.*

54. Calculate 112 oz. 17 dwt. 20 gr., at 4*s.* 7$\frac{1}{2}$*d.* per oz.—Ans. 26*l.* 2*s.* 1$\frac{13}{80}$*d.*

55. Calculate 236 oz. 12 dwt. 17 gr., at 12*s.* 6$\frac{1}{4}$*d.* per oz.—Ans. 148*l.* 12*s.* 8$\frac{121}{160}$*d.*

56. Calculate 18 lb. 11 oz. 19 dwt. 23 gr., at 4*s.* 5$\frac{1}{4}$ *l.* per oz.—Ans. 50*l.* 11*s.* 8$\frac{231}{1760}$*d.*

57. Calculate 3276 ac. 3 ro. 4 po., at 17*l.* 19*s.* 8*d.* per acre.—Ans. 58954*l.* 12*s.* 9$\frac{9}{10}$*d.*

58. Calculate 1264 hhd. 50 gall., at 40*l.* 17*s.* 5*d.* per hhd.—Ans. 51693*l.* 3*s.* 4$\frac{2}{3}$*d.*

59. 20 tons, 13 cwt. 3 qrs.,* at 1*l.* 4*s.* 2$\frac{1}{2}$*d.* per ton. —Ans. 25*l.* 0*s.* 9$\frac{2}{3}$$\frac{1}{2}$*d.*

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* If we reckon a pound the value of a ton, 1*s.* will be the value of a cwt. and 3*d.* of a qr. ∴ we say 20 tons, 13 cwt. 3 qrs., = 20*l.* 13*s.* 9*d.*; and then

		£	*s.*	*d.*
4*s.* =	$\frac{1}{5}$	20	13	9
2*d.* =	$\frac{1}{24}$	4	2	9
$\frac{1}{2}$*d.* =	$\frac{1}{4}$	0	3	5$\frac{1}{4}$
		0	0	10$\frac{1}{2}$
		£25	0	9$\frac{3}{4}$.—Ans.

ALLOWANCES ON GOODS

Gross Weight is the whole weight of any commodity, including that which contains it.

Tare Weight is the weight of the cask or box, &c. which contains any commodity.

Note. 1.—The weight of that which contains the commodity is the *real tare;* the established allowance for that weight is called the *customary tare;* an allowance for it at a given rate per cwt. or per cent. is called *proportionate tare;* and an allowance of a certain number of lbs. on each package, according to the ascertained average weight of a few, is called the *mean* or *average tare.*

Draft is an allowance per cask or bag, &c. on some commodities, that the weight may hold out when sold by retail.

Note 2.—Allowances were formerly made called *Tret and Cloff;* the former being 4 lb. on every 104 lb. or $\frac{1}{25}$ for dust or waste on goods retailed by the lb., and the latter 2 lb. on every 3 cwt. or $\frac{1}{168}$ for waste also: but these allowances are now generally discontinued.

Net weight is what remains after all allowances have been deducted.

To find the Net Weight.

Rule.—First subtract the draft (when any is allowed) from the gross weight; then find the tare on the remainder, by taking aliquot parts of 112 lbs. when it is at so much per cwt.; or of 100 lbs. when at so much per cent.; or by multiplication when it is at so much per cask, &c. on an average; or by Addition when the *real tare* of each cask or bag is given, which deducted from the gross weight when there is no draft, or from what remains after the draft is subtracted, when there is any, leaves the net weight.

Note 3.—If the tare, when computed, amount to any number of lbs. with a fraction less than $\frac{1}{2}$ lb., reject it; but, if it be more than $\frac{1}{2}$ lb., reject it, and increase the tare by 1 lb.

EXAMPLE.

What is the net weight of 7 bags of cotton wool, each weighing 2 cwt. 1 qr. 2 lb.; draft 1 lb. per bag, tare $2\frac{1}{2}$ lb. per cent?

	cwt.	qrs.	lbs.	
	2	1	2	
			7	
	15	3	14	gross weight.
			7	draft.
$2\frac{1}{2} = \frac{1}{40}$)	15	3	7	
	0	1	16	tare.
	15	1	19	net weight.—Ans.

EXAMPLES.

1. What if the net weight of 468 cwt. 3 qrs. 16 lb.; tare 14 lb. per cwt.?—Ans. 410 cwt. 1 qr. 3 lb.

2. What is the net weight of 896 cwt. 1 qr. 14 lb.; tare 8 lb. per cwt.?—Ans. 832 cwt. 1 qr. 11 lb.

3. What is the net weight of 514 cwt. 2 qrs. 8 lb.; tare 25 lb. per cent.?—Ans. 385 cwt. 3 qrs. 20 lb.

4. Find the net weight of 5 chests of souchong, each weighing, gross, 1 cwt. 1 lb.; draft, 1 lb., and tare 23 lb. per chest.—Ans. 3 cwt. 3 qrs. 25 lb.

5. Find the net weight and value of 5 bags of cotton wool, weighing 12 cwt. 2 qrs. 8 lb., deducting draft 1 lb. per bag, and tare $2\frac{1}{2}$ per cent., at 2*s.* 2*d.* per lb. —Ans. Net 1368 lb., and value £148 4*s.*

6. Find the net weight and value of 4 bales of goat's wool, weighing, gross, 15 cwt., 2 qrs. 1 lb.; draft 2 lb. per bale, tare 4 lb. per cwt. at 4*s.* 6*d.* per lb.—Ans. Net 1667 lb. and value £375 1*s.* 6*d.*

7. Find the net weight and value of 4 barrels of rice, weighing, gross, 11 cwt. 1 qr. 13 lb.; tare 2 cwt. 1 qr. 17 lb., draft 2 lb. per barrel, at 19*s.* 6*d.* per cwt.— Ans. Net, 8 cwt. 3 qrs. 16 lb., value £8 13*s.* $4\frac{1}{14}$*d.*

8. Find the net weight of 5 barrels of rice, weighing as follows: No. 1,—4 cwt. 4 lb.; tare 59 lb. No. 2,—3 cwt. 3 qr. 18 lb.; tare 66 lb. No. 3,—4 cwt. 1 qr. 14 lb; tare 62 lb. No. 4,—4 cwt. 1 qr. 18 lb; tare 53 lb. No. 5,—3 cwt. 3 qr. 11 lb.; tare 60 lb., and draft 2 lb. per barrel.—Ans. 17 cwt. 3 qr. 7 lb.

PROPORTION.

Definition.—PROPORTION is that rule by which a number is found, having to a given number the same *ratio* which is between two other given numbers; or any given pairs of numbers of the same kind.

Note 1.—The author* of an eminent work defines the *ratio* of two numbers to be the quotient of the greater divided by the less, as 4 : 12 = 3, and 12 to 4 = 3, also; but the one he denominates the *ascending ratio*, and the other the *descending ratio*. And a system, the joint production of two gentlemen† equally eminent with the former, informs us that the ratio of two numbers is the quotient arising from the division of the first by the second, as 4 : 12 = ⅓, or 12 : 4 = 3. Innumerable instances of diversity of opinion could be produced with regard to the definition of the term *ratio*; but it will sufficiently answer our present purpose to consider it as the quotient of the second divided by the first, as 4 : 12 = 3, or 12 : 4 = ⅓‡.

* This gentleman is the writer of the articles Prosody, Quantity, and Versification, in Dr. Rees's Cyclopædia.

† Davidson, of Burntisland, and Scott, of Edinburgh.

‡ M'Dougall, of Stirling Academy, in his work, defines it in this way.

Rule.—1. Put that number as the third term, which is of the same kind of thing with what is sought.

2. Then take any pair of numbers of the same kind, and consider, from the ratio which exists between them, whether the answer should be more or less than the third term.

3. If more, put the greater of the two numbers as the consequent or second term, and the less as the antecedent or first term; but, if the answer should be less than the third term, the contrary.

Remark.—1. This is all that is necessary in stating a question consisting of three numbers only.

4. The remaining pairs of the same kind, if any, must be managed precisely the same as the first, placing the antecedents and consequents perpendicularly under each other.

Note 2.—An antecedent is the first, and a consequent the last of two numbers compared with each other.

Remark 2.—The above method of arranging the terms supersedes the necessity of distinguishing proportion into direct and inverse, and simple and compound. It is a useless distinction, calculated only to perplex the learner, and consequently to retard his progress, as well as to subvert the simplicity of Proportion.

5. Next reduce the third term to the lowest name mentioned in it, and each pair of antecedents and consequents to the same denomination.

6. Then the third term multiplied by the product of the consequents, and this last product divided by the product of the antecedents, will give the answer in the same name or denomination in which the third term was left, which, if necessary, must be brought to the highest denomination of its kind. Or,

Rule 2.—Place the third term, and all the consequents, with the sign of multiplication between them above a horizontal line, and all the antecedents in the same manner under it; then proceed as in *Case* 4 of Arithmetical Fractions. This will produce the answer in the denomination in which the third term was left. Or,—

Rule 3.—The third term multiplied by the ratio of the first and second products will produce the answer.

EXAMPLE I.

If 144 lbs. of tea cost 60*l.* 17*s.* 6*d.* what should be given for 1008 lbs. at the same rate?

Method 1.—*Rule* 1.

lb.	lb.	£	s.	d.
144 :	1008 ::	60	17	6
		40		
		2435 sixp.		
		1008		
		19480		
		2435		
		12) 2454480		
144		12) 204540		
		4,0) 1704,5 sixp.		
		£426	2	6.—Ans.

Method 2.—*Rule* 2.

$$\frac{2435 \times 1008}{144} = \frac{2345 \times 84}{12} = \frac{2345 \times 7}{1} = 17045 \text{ sixp.} = £426\ 2s.\ 6d.\text{—Ans.}$$

Method 3.—*Rule* 3.

The ratio between 144 and 1008 is 7 ∴ £60 17*s.* 6*d.* × 7 = £426 2*s.* 6*d.*—Ans.

Remark 3.—This question belongs to the class of Direct Proportion.

Elucidation.—*Method* 1. As money is wanted for the answer, £60 17*s.* 6*d.* are placed as the third term; and, since 1008 lbs. must cost more than 144 lbs., 1008 are put as the consequent, and 144 as the antecedent;

the third term is reduced to its lowest name, that is, sixpences; the antecedent and consequent are already in the same name; afterwards the second and third terms are multiplied together, and the product divided by the first which produces the answer in sixpences, and these sixpences divided by 40 gives the answer in pounds.

Method 2.—The fractional form of this method evidently shews that the second and third terms are multiplied together, and their product divided by the first, and is managed as directed in the notes of *Case* 3 in Arithmetical Fractions. (See first *Case* 4, then *Case* 3, and afterwards *Case* 2.)

Method 3.—The ratio of 144 : 1008 = 7; therefore the third term multiplied by 7, agreeably to the third rule, gives the answer.

Note 3.—When the first and second or first and third terms are divisible by any number whatever without a remainder, the quotients, instead of the original numbers, may be used: for example, let 24 and 96 be either the first and second, or first and third; then 24 : 96 = 4 the ratio; now let us divide both by 12, and we have 2 : 8 = 4 the ratio, as before. Again, let us divide these quotients by 2, and we have 1 : 4 = 4 the ratio, as before. Now, since the ratio is the same in each case, it follows that *any* numbers having the same ratio will produce the same results; but, as small multipliers and divisors are more easily managed than those of greater magnitude, this abbreviated method is therefore preferable to the general one, as exhibited in *Method* 1.

Note 4.—In many cases, particularly when the second term is a number not greater than 144, the third term need not be reduced, as in *Method* 3.

Note 5.—The antecedent and consequent of each pair of terms are brought into the same name for the purpose of reducing them to simple numbers, which are more easily managed than compound ones.

Note 6.—The reason why the second and third terms are multiplied together, and their product divided by the first, may be very simply explained; thus—If 2 yards of cloth cost £1, what should be given for 24

yards? If £1 were the price of 1 yard, it is evident that £24 would be the price of 24 yards; but, as £1 is not the price of 1, but of 2 yards, half of £24, or £24 divided by 2, must therefore be the Answer.

It is obvious, then, from this demonstration, that 1 and 24 are multiplied together, and their product divided by 2, which is agreeable to the rule, and which will be seen by the following stating.

yds. yds. £
2 : 24 :: 1
1

2) 24

£12.—Ans.

But better thus:

yds. yds. £
2 : 24 :: 1

1 £12—Ans.

EXAMPLE II.

Lent my friend £500 7s. 6d. for 147 days, how long should he in return lend me £250 3s. 9d.?

£ s. d. £ s. d. da.
250 3 9 : 500 7 6 :: 147
20 20 2

5003 10007 294 da.—Ans.
4 4

20015 threep. 40030 threep.

4003 8006

1 2

Remark 4.—This question belongs to what is generally called Inverse or Reciprocal Proportion; but it is, more properly speaking, in *inverse ratio.*

Elucidation.—First, as days are wanted, 147 days are put as the third term; then, as I must have the use of a small sum longer than my friend had the use of a larger one from me, the greater is placed as the second term, and the less as the first; then they are both divided by 5, and afterwards by 4003, (per *Note* 3;) and, as the first term is reduced to unity, the number which stands in the second term is therefore the ratio; consequently, the third term multiplied by this will produce the answer, per *Rule* 3.

EXAMPLE III.

If 336 men, in 5 days of 10 hours each, dig a trench of 5 degrees of hardness, which is 70 yards long, 3 wide, and 2 deep, what length of trench, of 6 degrees of hardness, 5 yards wide, and 3 deep, may be dug by 240 men, in 9 days of 12 hours each?

Method 1.—*Note* 3.

ante.		conse.		long.
336 m.	:	240 m.	::	70 yards.
5 d.	:	9 d.		———
10 h.	:	12 h.		1
6 d.	:	5 d.		
5 w.	:	3 w.		
3 d.	:	2 d.		
15120,00	:	7776,00		
216		864		
24		72		
2		36 yards long.—Ans.		
1				

Method 2.—See Multiplication of Arithmetical Fractions.

$$\frac{\overline{70} \times \overset{4}{\overline{240}} \times \overline{9} \times \overline{12} \times \overline{5} \times \overline{3} \times \overline{2}}{\overline{336} \times \overline{5} \times \overline{10} \times \overline{6} \times \overline{5} \times \overline{3}} = 4 \times 9 = 36 \text{ yds.—Ans.}$$

$$\overline{28}$$

$$\overline{2}$$

Method 3.

$$77760 \div 1512000 = \frac{7776}{15120} = \frac{1944}{3780} = \frac{486}{945} = \frac{54}{105}$$

$$= \frac{18}{35} \text{ the ratio;}$$

Then $\overline{70 \times 18} \div 35 = 2 \times 18 = 36$ yds.—Ans.

Remark 5.—This is what is denominated Compound Proportion.

Elucidation.—As length is wanted, 70 yards are put as the third term; then the first pair of terms of the same kind is 336 and 240 men; now, if 336 men dig 70 yards, 240 men will dig less; ∴ 240 being the less number, is put in the second term, and 336 in the first: the second pair is 5 and 9 days; if 5 days produce 70 yards, 9 days will produce more; ∴ 9, being the greater number, is put as the second term: the third pair is 10 and 12 hours; if 10 hours produce 70 yards, 12 will produce more; ∴ 12 is put as the second term: the fourth pair is 5 and 6 degrees of hardness; when the earth is 5 degrees of hardness, 70 yards are produced; when it is a degree harder, less work can be done; ∴ 5 is put on the second term: the fifth pair is 3 and 5 wide; if 3 wide require 70 long, 5 wide will require less in length; ∴ 3 is put in the second term: the last pair is 2 and 3 deep; if 2 deep require 70 long, 3 deep will require less; ∴ 2 is put in the second term.

And, agreeably to *Rule* 1, the third term multiplied by the product of the consequents, and divided by the product of the antecedents, would have produced the answer; thus, $\overline{777600 \times 70} \div 1512000 = 36$. But we have preferred the abbreviating method, and therefore

have employed 100 as the divisor of the first and second terms; then 70 for the first and third; and afterwards 9, 12, and 2, for the first and second. As the first and third terms are reduced to unity, the second or middle term is therefore the answer. Had the first and second been 1, the third term would also have been the answer; or should they be any two numbers whatever, provided they be the same, the same result will follow: for, as the first term is a divisor, and the second and third multipliers, it follows that when the first and either of the other two terms are the same, the remaining term must be the answer; for, if we increase any number in any ratio whatever, and reduce its increase by the same ratio, it must necessarily be exactly what it was before any such Multiplication or Division was employed.

The second and third methods will be very easily understood by consulting first *Case* 4 and then *Case* 3 in Arithmetical Fractions.

Remark 6.—It is clearly evident, from the above methods of solving this question, which consists of 13 terms, that Compound Proportion is just as easily worked as Simple Proportion: this is the only reason for the author having included both under the general term—PROPORTION.

EXAMPLES.

1. If 2 lb. of tea cost 16*s*. what cost 6 lb.?*—Ans. £2 8*s*.
2. If 6 lb. of tea cost £2 8*s*. what cost 2 lb.?—Ans. 16*s*.
3. If 16*s*. be the price of 2 lbs. of tea, how many pounds can I get for £2 8*s*.?—Ans. 6 lb.
4. If I pay £2 8*s*. for 6 lb. of tea, how many lbs. can I have for 16*s* ?—Ans. 2 lb.
5. If 3 lbs. of bohea cost 27*s*. 6*d*. what is the value of 18 lbs.?—Ans. £8 5*s*.

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* The three following questions are deduced from this. Every example admits of being varied in the same manner; it would, therefore, be a very good exercise for the pupil to vary each example, at least the greater part of them, in the same way.

6. If I pay 5*s*. 6*d*. for 7 lb. of sugar, what is the price of 1 cwt.?—Ans. £4 8*s*.

7. If 12 lbs. of cinnamon cost 18*s*. 4*d*. what cost 84$\frac{1}{2}$ lbs.?—Ans. £6 9*s*. 1$\frac{1}{2}$*d*.

8. If a cwt. of cheese cost £5 10*s*. 6*d*. what is the price of a stone?—Ans. 13*s*. 9$\frac{1}{4}$*d*.

9. If a piece of dimity of 56 yards cost £5 18*s*. what must be given for 29$\frac{3}{4}$ yards?—Ans. £3 2*s*. 8$\frac{1}{4}$*d*.

10. If a piece of print measuring 19$\frac{1}{4}$ yards cost £8 13*s*. 9*d*. what cost 1350$\frac{1}{4}$ yards? — Ans. £593 18*s*. 9$\frac{39}{77}$*d*.

11. If the expense of house-keeping for 7$\frac{1}{7}$ weeks be £40 13*s*. 8$\frac{1}{2}$*d*. what is the yearly outlay, allowing 52$\frac{1}{7}$ weeks to the year?—Ans. £297 0*s*. 2$\frac{17}{20}$*d*.

12. The house-keeper's wages are 50 guineas per year; she has been in the family from January 8th till October 26th, 1824; what have I to pay her? —Ans. £42.

13. If 24 men cut down 32 acres of corn in 18 days, how many men will it require at that rate to cut down 128 acres in 36 days?—Ans. 48 men.

14. If 24 men cut down 128 acres of corn in 36 days, in how many days will 12 men cut down 32 acres at that rate?—Ans. 18 days.

15. If 24 men cut down 128 acres in 36 days, how many acres will 12 men cut down at that rate in 18 days?—Ans. 32 acres.

16. If 8 horses require 40 guineas' worth of hay in 6 months, when hay sells at 8*d*. per stone, how much will it require to maintain 7 horses 11 months, when hay is as low as 5*d*. per stone?—Ans. £42 2*s*. 2$\frac{1}{2}$*d*.

17. If an iron bar, 4 feet long, 3 inches broad, and 1$\frac{1}{2}$ inch thick, weigh 36 lb. how much will a bar weigh that is 6 feet long, 4 inches broad, and 2 inches thick? —Ans. 96 lbs.

18 If a person travel 12 hours a day, and finish his journey in three weeks, how long would the same journey take him if he travelled only 9 hours a-day? —Ans. 4 weeks.

19. Three outlets to a large cask will empty it in 30 minutes; how many such outlets will perform the same thing in 3$\frac{1}{4}$ minutes?—Ans. 24 outlets.

20. If £756 17*s*. 6*d*. as a principal, gain

£37 16*s*. 10½*d*. in 12 months, what principal will gain the same sum in 3¾ months?—Ans. £2522 18*s*. 4*d*.

21. How much souchong tea, at 7*s*. 6*d*. per lb. must be given in exchange for a cwt. of congou, at 6*s*. 8*d*.? —Ans. 99⅝ lbs.

22. How many men must be employed to perform a piece of work in 2 days that 3 men can do in 6?—Ans. 9 men.

23. As 12 inches long, 12 broad, and 12 deep, make a solid foot, what length of timber which is 7 inches broad and 3 inches thick, will constitute a solid foot?—Ans. 82⅞ in.

24. If 16 oxen be at grass 3 weeks for £9 6*s*. 8*d*., how many oxen may graze on the same pasture 12 weeks for £18 13*s*. 4*d*.?—Ans. 8 oxen.

25. If 21 apples are of the same value as 29 pears, how many apples are worth 87 pairs?—Ans. 63 apples.

26. How much land will be requisite to produce as much corn, when the fertility of the soil yields 100 fold, as was produced on 20 acres when the seed yielded 60 fold?—Ans. 12 acres.

27. If 300 men in 6 days can build a fortification 450 yards long, 9 yards wide, and 6 feet high, in how many days will 425 men construct a rampart 1200 yards long, 12 yards wide, and 12 feet high?—Ans. $30\frac{2}{17}$ days.

28. If the freight of a steam-boat of 360 tons burthen, for 8 months, be £736, what should be paid for the hire of a similar vessel of 270 tons for 12 months? —Ans. £828.

29. How long should A accommodate B with the loan of £739, who lent A £1200 for 130 days?—Ans. $211\frac{71}{739}$ days.

30. Three valves will exhaust a reservoir in 1¼ hour; how many such valves will empty it in 15½ minutes?—Ans. $20\frac{10}{11}$ valves.

31. How many yards of cloth may be purchased for 80½ guineas, at the rate of 5*s*. 3*d*. per ell English?—Ans. 402 yds. 2 qrs.

32. If a ball of 18 lb. be shot from a cannon with such a force as to send it 100 feet in a second, with what velocity would a ball of 24 lb. move were it impelled by the same force?—Ans. 75 feet.

33. There are two equal parallelograms;* the length of the one is 10 feet, 6 inches, and its breadth 7 feet, 3 inches; the breadth of the other is 4 feet, 2 inches: what is its length?—Ans. 18 feet, $3\ \frac{6}{13}$ inches.

34. If the sixpenny loaf weigh 4 lbs. when flour is at 2*s*. per peck, what should be its weight when flour rises to 2*s*. 9*d*. per peck?—Ans. 2 lb. 14 oz. $8\frac{8}{11}$ dr.

35. There is a monument which throws a shadow of 240 feet, at the same time that an elevated staff of 3 feet, 4 inches, throws a shadow of 6 feet: what is the height of the monument?—Ans. 133 ft. 4 in.

36. The battering ram of Vespasian weighed 100,000 lb. and was moved, let us admit, with such a velocity as to pass through 20 feet in 1 second of time; and this was found sufficient to demolish the walls of Jerusalem. With what velocity must a cannon ball, that weighs but 30 lbs. be moved in order to do the same execution?—Ans. 66666 ft. 8 in. per sec.

37. Suppose a person to travel 152 miles in 7 days, when the days are 12 hours long; how many days will he be in travelling 576 miles when the days are 16 hours long?—Ans. $19\frac{1}{19}$ days.

38. Sold $328\frac{3}{4}$ yards of cloth for £317 $\frac{1}{8}$: how much should I receive for $26\frac{3}{8}$ yards?—Ans. £25 9*s*. $10\frac{2}{5}\frac{9}{5}\frac{3}{5}\frac{3}{5}d$.

39. If 1080 men consume 42 hhds. of wine in 3 weeks, at the rate of $\frac{1}{4}$ of a pint to each man per day, how long will 500 hhds. serve 360 men, at the rate of $1\frac{1}{8}$ pint to each man per day?—Ans. 58 wks. $3\frac{1}{11}$ da.

40. If $\frac{7}{11}$ of an ell English cost 10*s*. $2\frac{2}{11}d$. what will $\frac{4}{5}$ of a yard cost?—Ans. 8*s*.

41. If $\frac{3}{5}$ of $\frac{7}{9}$ of a yard cost $\frac{4}{5}$ of $\frac{6}{7}$ of a pound, what will 179 ells English cost?—Ans. £199 15*s*. $6\frac{4}{7}d$.

42. If I have $3\frac{3}{4}$ cwt. carried $15\frac{1}{2}$ miles for 4 guineas, how far should $9\frac{1}{2}$ cwt. be carried for 20 guineas?—Ans. $30\frac{4}{7}\frac{2}{3}$ miles.

43. If 248 men in $5\frac{1}{2}$ days, of 11 hours each, dig a trench of 7 degrees of hardness, $232\frac{1}{2}$ yards long, $3\frac{1}{3}$ wide, and $2\frac{1}{3}$ deep, in how many days of 9 hours long will 24 men dig a trench of 4 degrees of hardness, $337\frac{1}{2}$ yards long, $5\frac{3}{4}$ wide, and $3\frac{1}{2}$ deep?—Ans. $185\frac{1}{4}\frac{5}{6}$ da.

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* A parallelogram is a right-lined quadrilateral (or figure of four sides) whose opposite sides are parallel and equal.

44. I agreed to pay 075 guinea for the carriage of 2.5 tons of goods 2.9 miles: what is that per cwt. for a mile?—Ans. $\frac{174}{725}$ farthing.

45. If 4 compositors, in 16 days, of 12 hours long, can compose 14 sheets of 24 pages each sheet, 44 lines in each page, and 40 letters in a line; in how many days, of 10 hours long, may 9 compositors compose a volume consisting of 30 sheets, 16 pages in a sheet, 48 lines in a page, and 45 letters in a line?—Ans. $14\frac{14}{77}$ days.

46. If 4 compositors, working 12 hours for 5 days in each week, can in 9 weeks compose 3 volumes of 10 sheets each, 24 pages being in each sheet, 44 lines in a page, and 40 letters in a line; in how many weeks, at 9 hours, for 6 days each, may 14 compositors set up 7 volumes of 15 sheets, of 16 pages each, having 48 lines in a page, and 55 letters in a line?—Ans. 10 weeks.

Remark 7.—Questions in Proportion may very frequently be solved mentally; thus, 1. If 12 persons spend £240 in 8 months, how much should 18 persons spend in 4 months? If 12 persons spend £240, 18 persons may spend a half more ∴ £240 + 120 = £360; but 4 months' expenditure must only be half of 8 months' ∴ £360 ÷ 2 = £180.—Ans.

Or $\frac{240}{8}$ = £30 expended per month by 12 persons.

$\frac{1}{2}$ of 30 = $\frac{15}{\therefore 45}$ expended per month by 18 ditto.

And ∴ 45 × 4 = £180 spent by 18 in 4 mo.—Ans.

2. How many cwts. may be carried 4 miles for 10*s*. if 10 cwt. can be carried 8 miles for 5*s*.? If 5*s*. pay the carriage of 10 cwt. 8 miles, it is evident that 10*s*. will pay for 20 cwt. the same distance, namely, 8 miles, or 40 cwt. 4 miles. The answer is ∴ 40 cwt.

3. If 10 labourers, working four hours a day, do a piece of work in 3 days, how long will 6 labourers, working 5 hours a day, take to do it?—If 10 labourers do it in 3 days, 30 will in 1 day; and, if 30 do it in 1 day of 4 hours, 120 will finish it in 1 day of 1 hour, ∴

I require 20 days of 1 hour each to do the of 5 hours each they will be able to accomplish it in $\frac{1}{4}$ of 20 days, which is 4 days:—Ans.

DISTRIBUTIVE PROPORTION.

Definition.—Distributive Proportion is that rule by which the stock, loss, or gain, of two or more persons jointly concerned is divided in proportion to the share of each. But, when their stocks have been employed unequal times, the stock, loss, or gain, is divided in proportion to the complex ratio of each share, and the time of its continuance.

Case 1.—When the capital is divided into different shares, to find each respective gain or loss.

Rule.—Divide the gain or loss by the number of shares; this quotient multiplied by the shares each partner holds, will give his share of the gain or loss.

EXAMPLE.

Watson, Watts, Wilkinson, and Williams, purchase the ship *Georgina*, consisting of 18 shares; Watson bought 6 shares, Watts 7, Wilkinson 3, and Williams 2. They receive of net freight for a voyage £315 14*s.* 6*d.* How much of this sum ought each to receive?

		£	*s.*	*d.*	
18	6	315	14	6	
	3	52	12	5	
		17	10	$9\frac{3}{5}$	× 6 × 7 × 3 × 2
		105	4	10	Watson's share.
		122	15	$7\frac{2}{5}$	Watts's ditto.
		52	12	5	Wilkinson's ditto.
		35	1	$7\frac{1}{5}$	Williams's ditto.
		£315	14	6	Proof.

Case 2.—When each particular stock, and the gain or loss, are given, to find each respective share of that gain or loss.

Rule.—As the total stock is to the stock of each partner, so is the total gain to the gain of each.

EXAMPLE.

Three merchants, A, B, and C, enter upon a joint adventure; A advances £150, B £300, and C £450. They make a clear gain of £400: what is each person's share of this gain?

150 + 300 + 450 = £900 total stock.

		£	s.	d.	
900 : 150 :: 400 :		66	13	4	A's share.
900 : 300 :: 400 :		133	6	8	B's share.
900 : 450 :: 400 :		200	0	0	C's share.
	£400	0	0	Proof.	

Note 1.—As the above method would be rather tedious when the partners are numerous, it is preferable to divide the whole gain or loss by the total stock, and the quotient will be a common multiplier for each partner's stock; thus,

$\frac{400}{900} = \frac{4}{9}$, common multiplier.

	£	s.	d.	
$\therefore \frac{150 \times 4}{9} =$	66	13	4	A's share, as before.
$\frac{300 \times 4}{9} =$	133	6	8	B's share.
And $\frac{450 \times 4}{9} =$	200	0	0	C's share.
£400	0	0.	—Proof.	

Case 3.—When the dividend is to be proportioned to the time the capital is employed.

Rule.—As the sum of the products of the several

capitals into the times they have been respectively employed is to each particular product, so is the whole dividend to each individual share.

EXAMPLE.

A deposits £100 for 5 months, B £120 for 4 months, and C £140 for 3 months; they gain £120; required the share of each?

A = 100 × 5 = 500
B = 120 × 4 = 480
C = 140 × 3 = 420

1400

				£	s.	d.	
Then 14	: 5 ::	120	:	42	17	1½	A's share.
140	: 48 ::	120	:	41	2	10½	B's ditto.
140	: 42 ::	120	:	36	0	0	C's ditto.
				£120	0	0	Proof.

EXAMPLES TO CASE 1.

1. Divide a guinea amongst three persons, in the proportion of 4, 2, and 1.—Ans. 12*s.*, 6*s.*, and 3*s.*

2. Divide a sovereign amongst 5 persons, in the proportion of 6, 5, 4, 3, and 2.—Ans. 6*s.*, 5*s.*, 4*s.*, 3*s.*, and 2*s.*

3. Let the least common multiple* of the 9 digits be divided into 9 such parts that shall be to each other as 1, 2, 3, 4, 5, 6, 7, 8, 9, respectively.—Ans. 56, 112, 168, 224, 280, 336, 392, 448, and 504.

EXAMPLES TO CASE 2.

4. Three merchants, W, A, and R, freight a ship with wine; W puts on-board 350 tons, A 500, and

* To find the least common multiple, see Reduction of Arithmetical Fractions.

R 250: in a storm, 180 tons were thrown overboard: what loss does each sustain?—Ans. 57$\frac{3}{11}$ W's loss 81$\frac{2}{11}$ A's; and 40$\frac{10}{11}$ R's.

5. W, A, and R, buy a ship for £1750; of which W paid £840, A £485, and R the rest. The net freight for a voyage is £145 15*s.*: how much of this sum should each receive?—Ans. W. £69 19*s.* 2$\frac{8}{11}$*d.*; A £40 7*s.* 10$\frac{6}{11}$; and R £35 7*s.* 11$\frac{1}{11}$*d.*

6. Four partners, A, B, C, and D, by several losses, found their stock reduced to £1600; in consequence of which they dissolved partnership: in what proportion shall their diminished capital return to them, since A originally advanced £540, B £250, C £300, and D £1000?—Ans. A £413 7*s.* 11$\frac{45}{109}$; B £191 7*s.* 9$\frac{3}{109}$*d.*; C £229 13*s.* 3$\frac{120}{109}$*d.*; D £765 11*s.* 0$\frac{12}{109}$*d.*

EXAMPLES TO CASE 3.

7. E, L, I, Z, and A, embark in business; E puts in, as capital, £750 for 3 months; L £785 for 4 months; I £1115 for 5 months; Z £7175 for 6 months; and A £730 for 7 months: their gain is £2300: what is each individual's share of it?—Ans. E £87 10*s.* 6$\frac{4}{13}$*d.*; L £122 2*s.* 11$\frac{4}{13}$*d.*; I £216 17*s.* 5$\frac{2}{13}$*d.*; Z £1674 13*s.* 5$\frac{6}{13}$*d.*; and A £198 15*s.* 7$\frac{10}{13}$*d.*

8. A, B, C, and D, rent a grass inclosure, for which they agree to pay £80. A put in 18 cattle for 100 days; B 16 cattle for 120 days; C 20 cattle for 98 days; and D 7 cattle for 280 days: how much of the rent should each pay?—Ans. A £18 16*s.* 11$\frac{10}{191}$*d.*; B £20 2*s.* 1$\frac{28}{191}$*d.*; C £20 10*s.* 5$\frac{126}{191}$*d.*; D £20 10*s.* 5$\frac{126}{191}$*d.*

9. A family of 8 persons took a large house for £100 per annum; at the end of half the year they took 3 lodgers, and 4 weeks after 3 more, and so on for every successive 4 weeks during the remainder of the term: what must one of each class pay per week of the rent?—Ans. 2*s.* 9$\frac{7}{11}$*d.*

10. In an Arabic manuscript was found the following remarkable decision:—Two Arabians sat down to dinner; one had 5 loaves, and the other 3: a stranger, passing by, desired permission to eat with them, to which they agreed. The party having finished, the stranger laid down 8 pieces of money, and departed.

The proprietor of the 5 loaves took up 5 pieces, and left 3 for the other, who objected, and insisted on half. The cause came before Ali, the chief magistrate, who gave the following judgment:—Let the owner of the 5 loaves have 7 pieces, and the owner of the 3 loaves 1. Was this decision right?—Ans. Yes.

11. Four persons met at the foot of Benlomond, and purposed reaching its summit before sun-rise, that they might have the pleasure of seeing the bright luminary rise with majestic grandeur: A had 5 pints of Highland whiskey with him, B $4\frac{1}{2}$, C $3\frac{1}{2}$, and D (who was an expert accountant,) told the others that he had no whiskey, but that, if they allowed him to partake equally with themselves, that his exact proportion would be 10*s.* 10*d.*: this they agreed to, but quarrelled about the division of the money; it was therefore deferred to the decision of D, who gave A 5*s.* 10*d.* B 4*s.* 2*d.*, and C 10*d.* Was it divided justly?—Ans. Yes.

12. Three persons traded together; A's stock was £89 5*s.*, B's £92 15*s.*, and C's £38 10*s.*; their respective gains were £25 10*s.*, £37 2*s.*, and £24 4*s.*: also, if the times that each person's stock was employed in trade be added together, the sum will be 23 months. How long was each man's stock in trade?*—Ans. A 5 mo., B 7 mo., C 11 mo.

SIMPLE INTEREST.

Definition.—1. Simple Interest is an allowance given by the borrower to the lender of money, at a certain rate per cent. for a definite space of time.

* Reduce each man's stock and gain into one denomination, and multiply each man's gain into all the stocks except his own. Then, as the sum of the products is to each man's product, so is the sum of the times to the time of each.

2. *The principal* is the sum lent.

3. *The rate per cent.* is what the borrower agrees to pay for the use of each £100 for a year.

4. *The amount* is the sum of the principal and interest.

Case 1.—For Years.

Rule 1.—Multiply the principal by the rate and time, and divide by 100. Or,

2. Take aliquot parts of 100 for the rate, and multiply by the time. Or,

3. Multiply the rate and time together, and take aliquot parts of 100 for the product. Or,

4. 100 : principal :: rate multiplied by the time to the interest required. Or,

5. Multiply the principal by the years, and divide by the quotient of 100 divided by the rate.

EXAMPLE.

What is the interest of £500 for 3 years, at 5 per cent. per annum?

By Rule 1.

£
500
5
2500
3
£75.00.—Ans.

By Rule 2.

£
5 = 1/20) 500
25
3
£75.—Ans.

By Rule 3.

3 × 5 = 15

£
10 = 1/10) 500
5 = 1/2) 50
25
£75.—Ans.

By Rule 4.

100 : 500 :: 5 × 3 : £75.—Ans.

By Rule 5.

$100 \div 5 = 20.$

£500
3

2,0) 150,0

£75.—Ans.

Case 2.—For months.

Rule 1.—Multiply the principal by the number of months, and the product is the answer in pence, when the rate is 5 per cent. Or,

2. Reckon a penny for every pound of the principal for each month for the interest at 5 per cent. Or,

3. Of as many shillings as there are pounds in the principal take aliquot parts for the number of months, when at 5 per cent. Or,

4. Multiply the principal by the number of months, and divide by the quotient of 1200 divided by the rate for the interest at any given rate.

EXAMPLE.

Find the interest of £500 for 6 months, at 5 per cent. per annum.

By Rule 1.

£500
6

12) 3000 pence.

2,0) 25,0

£12 10.—Ans.

By Rule 2.

4,0) 50,0 sixp.

£12 10.—Ans.

By Rule 3.

6 = ½) 500 shil.

2,0) 25,0

£12 10.—Ans.

By Rule 4.

1200 ÷ 5 = 240

500
6

24,0) 300,0

£12 10.—Ans.

Case 3.—For weeks.

Rule 1.—Having found the interest for 1 year, say 52 weeks : the given weeks :: the interest of 1 year to the interest required. Or,

2. Having found the interest for 1 year, take aliquot parts of 52 for the given weeks. Or,

3. Multiply the principal by the given weeks, and divide by the quotient of 5200 divided by the rate.

EXAMPLE.

What is the interest of £500 for 39 weeks, at 5 per cent. per annum?

By Rule 1.

£
1/10) 500

25 int. for 1 year.

wks. wks. £ £ s.
Then 52 : 39 :: 25 : 18 15.—Ans.

By Rule 2.

£
26 wks. = ½) 25 int. for 1 yr.

13 wks. = ¼) 12 10
6 5

£18 15.—Ans.

By Rule 3.

5200 ÷ 5 = 1040

£500
39

1040) 19500

£18 15.—Ans.

Case 4.—For days.

Rule 1.—Multiply the principal by the days and by twice the rate, and divide by 73000. Or,

2. 365 : given days :: 1 year's interest to the interest required. Or,

3. Multiply the principal by the days, and divide by 7300 for the interest, at 5 per cent. Or,

4. Multiply the principal by the rate and days, and divide by 36500 for the interest, at any given rate. Or,

5. Multiply the principal by the days, and divide by the quotient of 36500 divided by the rate.

EXAMPLE.

What is the interest of £500 for 146 days, at 5 per cent. per annum?

By Rule 1.

£500
146

73000
10 twice the rate.

73,000) 730,000

£10.—Ans.

By Rule 2.

5 = $\frac{1}{20}$) 500

£25 = 1 yr's int.

da. da. £ £
365 : 146 :: 25 : 10.—Ans.

By Rule 3.

£500
146

73,00) 730,00

£10.—Ans.

By Rule 4.

£500
5

2500
146

365,00) 3650,00

£10.—Ans

By Rule 5.

36500 ÷ 5 = 7300.

£500
146

73,00) 730,00

£10.—Ans.

Note.—Interest for days at 5 per cent. may be found by the following *approximating* method:—Multiply the principal by the days; divide this product by 3, and repeat the quotient 3 times; set each succeeding quotient a figure more to the right; find their sum, and point off 4 places for decimals, independently of all others that may occur in the calculation.

Thus, £500
146

3) 73000
24333.3
2433.3
243.3

10.0010. = £10.—Ans.

EXAMPLES TO CASE 1.

1. What is the interest of £750 for 1 year, at 5 per cent. per annum?—Ans. 37*l.* 10*s.*

2. What is the interest of 1000*l.* for 3 years, at 4 per cent. per annum?—Ans. 120*l.*

3. What is the amount of 1273*l.* for 5 years, at 5 per cent. per annum?—Ans. 1591*l.* 5*s.*

4. What is the amount of 189*l.* 10*s.* for $5\frac{1}{2}$ years, at 4 per cent. per annum?—Ans. 231*l.* 3*s.* $9\frac{3}{5}d.$

5. What is the interest of 128*l.* 17*s.* 6*d.* for $3\frac{1}{4}$ years, at $5\frac{1}{2}$ per cent. per annum?—Ans. 26*l.* 11*s.* $7\frac{5}{11}d.$

6. What is the interest of 1256*l.* for $17\frac{3}{4}$ years, at $5\frac{3}{8}$ per cent. per annum?—Ans. 1176*l.* 12*s.* $7\frac{1}{2}\frac{1}{3}d.$

EXAMPLES TO CASE 2.

7. What is the interest of 800*l.* for 10 months, at 4 per cent. per annum?—Ans. 23*l.* 13*s.* 4*d.*

8. What is the interest of 926*l.* for 9 months, at $3\frac{1}{2}$ per cent. per annum?—Ans. 26*l.* 2*s.* $1\frac{1}{2}d.$

9. What is the amount of 839*l.* 10*s.* for $7\frac{1}{2}$ months, at $1\frac{1}{2}$ per cent. per annum?—Ans. 847*l.* 7*s.* $4\frac{1}{8}d.$

10. Find the amount of 1*l.* for $3\frac{1}{3}$ months, at 1 per cent. per annum?—Ans. 1*l.* 0*s.* $0\frac{1}{3}d.$

11. Find the interest of 73*l.* for 7, 8, and 9 months, at $2\frac{1}{2}$ per cent. per annum?—Ans. 1*l.* 1*s.* $3\frac{1}{2}d.$, 1*l.* 4*s.* 4*d.*, and 1*l.* 7*s.* $4\frac{1}{2}d.$

EXAMPLES TO CASE 3.

12. Find the interest of 184*l.* for 13 weeks, at 5 per cent. per annum?—Ans. 2*l.* 6*s.*

13. Find the interest of 326*l.* for 26 weeks, at 4 per cent. per annum?—Ans. 6*l.* 10*s.* $4\frac{4}{5}d.$

14. Find the amount of 730*l.* for 39 weeks, at $3\frac{1}{2}$ per cent. per annum?—Ans. 749*l.* 3*s.* 3*d.*

15. Find the amount of 132*l.* 10*s.* for $6\frac{1}{2}$ weeks, at $3\frac{1}{2}$ per cent. per annum?—Ans. 133*l.* 0*s.* $4\frac{7}{3}\frac{1}{2}d.$

EXAMPLES TO CASE 4.

16. What is the interest of 700*l.* for 35 days, at 5 per cent. per annum?—Ans. 3*l.* 7*s.* $1\frac{3}{7}\frac{5}{3}d.$

17. What is the interest of 526*l.* for 192 days, at 4 per cent. per annum?—Ans. 11*l.* 1*s.* $4\frac{41}{182}\frac{6}{5}d.$

18. Find the amount of 7300*l.* for 1 day, at 1 per cent. per annum?—Ans. 7300*l.* 0*s.* 4*d.*

19. Find the amount of 300*l.* for 1 day, at $7\frac{1}{2}$ per cent. per annum?—Ans. 300*l.* 1*s.* $2\frac{1}{3}\frac{1}{5}d.$

20. Find the interest of 27*l.* 10*s.* from January 7th till August 18th, at 5 per cent. per annum?—Ans. 1*l.* 6*s.* $9\frac{3}{5}d.$

21. Find the amount of 174*l.* 16*s.* 8*d.* from February 4th, 1824, till December 18th of the same year, at $7\frac{1}{2}$ per cent. per annum?—Ans. 186*l.* 5*s.* $1\frac{1}{3}\frac{1}{5}d.$

22. What is the amount of 500*l.* which lay at interest from March 8th, 1820, till December 25th, 1824, interest at $1\frac{1}{2}$ per cent.?—Ans. 536*l.*

23. A person let out a certain sum of money at 4 per cent. which in 12 years wanted but 20*l.* of the principal. What was the principal?—Ans. $38\frac{6}{13}l.$

COMPOUND INTEREST.

Definition.—Compound Interest is that which arises not only from the original principal, but also from the interest itself becoming an additional principal at the end of each succeeding payment.

Rule 1.—Find the amount of the given principal for the time of the first payment as in Simple Interest; then consider this as the principal for the second payment, and calculate the amount of this new principal, as before; this repeated as often as there are payments will be the amount required. Or,

2. Multiply the amount of £1 for the time of the first payment, by itself, for the second amount; this

multiplied by the first amount will give the third; this multiplied again will give the amount of the fourth payment, and so on till you have as many amounts as are denoted by the number of payments: this multiplied by the given principal will be the required amount.

Note 1.—The second amount multiplied by itself produces the fourth; the fourth multiplied by itself gives the eighth, &c. When the number of payments are many, this is the best method of procedure.

EXAMPLE.

What is the amount of £700 for 8 years, at 5 per cent. per annum, compound interest?

By Rule 1.

5 = $\frac{1}{20}$	700	
	35	
$\frac{1}{20}$	735	= 1st year's amount.
	36.75	
$\frac{1}{20}$	771.75	= 2d ditto.
	38.5875	
$\frac{1}{20}$	810.3375	= 3d ditto.
	40.516875	
$\frac{1}{20}$	850.854375	= 4th ditto.
	42.542718	
$\frac{1}{20}$	893.397093	= 5th ditto.
	44.669854	
$\frac{1}{20}$	938.066947	= 6th ditto.
	46.903347	
$\frac{1}{20}$	984.970294	= 7th ditto.
	49.248514	
	£1034.218808	=8th ditto.—1034*l.* 4*s.* 4½*d.* Ans.

By the Note.

1.05 = 1 year's amount of £1.
1.05

525
105

1.1025 = 2 years' amount.
1.1025

55125
22050
121275

1.21550625 = 4 years' amount.
526055121

1215506
243101
12155
6078
608
7

1.477455 = 8th year's amount, retaining 6 decimal
700 places. See Mult. of Decimals.

1034.2185 = £1034 4*s.* 4½*d.* Ans.

Remark 1.—The above rules will be true, whether the payments be yearly, half-yearly, quarterly, monthly, &c. but there must be a complete integral number of the times of payments; you cannot, therefore, find the interest payable *yearly* for 4½ years or 3¾ years, &c. by the above rules. But,

Note 2.—If the payments be half-yearly, take half the rate and twice the number of years; for quarterly payments take ¼ of the rate and 4 times the number of years, &c.—The truth of this note will easily appear to those who are acquainted with logarithmical calculations. By logarithms, the fractional and integral times may be very expeditiously computed.

Remark 2.—In short periods of time, simple and compound interests differ very little; thus, the simple interest of £700 for 4 years, at 5 per cent. is £140: the compound interest £150 17*s.* 1*d.*, a difference only of £10 17*s.* 1*d.* But when interest is allowed to accumulate for any considerable period, the increase, according to the method of Compound Interest, is almost beyond credibility. It, however, becomes more apparent if we consider that every 14¼ years the amount is doubled at 5 per cent. compound interest; consequently, £1 will, 14¼ years hence, amount to £2; the next 14¼ years to £4; and the next to £8; and so on, doubling every period. We cannot contrast simple and compound interests better than by the following observation, viz. that one farthing at simple interest from the commencement of the christian æra till the end of the year 1820, would make no more than 2*s.* 3¼*d.*; but at compound interest for the same time it would amount to no less a sum than 382166 quintillions, 904172 quadrillions, 243507 trillions, 264759 billions, 658264 millions, 749515 pounds, 12 shillings, and 11 pence!!* But this sum is so immensely great, that it is impossible for the human mind to comprehend it; it exceeds 156,718121 globes of solid gold, each as large as our earth!! and if these globes were to form a circular ring, mutually touching each other, the radius of this ring would be 198966,029181 miles. Now, the mean distance of the Georgium Sidus, from the centre of gravity of the solar system is 1800,000000 miles; hence, 198966,029181 divided by 1800,000000, gives 110½, that is the radius; and, consequently, the circumference of this ring would be 110½ times greater than the orbit of the Georgium Sidus!!

EXAMPLES.

1. What is the compound interest of 629*l.* for 3 years, at 5 per cent.?—Ans. 98*l.* 2*s.* 11$\frac{1}{100}$*d.*

* This calculation, which was a very irksome one, was performed about a twelvemonth ago, by Mr. SCOTT, an eminent mathematician, and pupil of the great PLAYFAIR, as well as by the author. It was afterwards inserted in the "Gleaner" of March.

2. Find the compound interest of 754*l.* 18*s.* 6*d.* for 5 years, at 4 per cent.?—Ans. 163*l.* 11*s.* 1½*d.*

3. What is the compound interest of 800*l.* for 4 years, at 2½ per cent.?—Ans. 83*l.* 1*s.* $0\frac{3}{25}d.$

4. What is the amount of 174*l.* 10*s.* for 6 years, at 2⅔ per cent.?—Ans. 201*l.* 3*s.* 8¼*d.*

5. Required the compound interest of 700*l.* for 8 years, at 5½ per cent.?—Ans. £361 13*s.* 10½*d.*

6. Required the amount of 500*l.* for 10 years, at 10 per cent.?—Ans. 1296*l.* 17*s.* 7½*d.*

7. To what sum will 1000*l.* amount to in 20 years, at 5 per cent.?—Ans. 2653*l.* 6*s.*

8. Mr. Farquhar purchased Fonthill Abbey lately for 330,000*l.*: what is the compound interest of this sum for 8 years, at 4½ per cent.?—Ans. 139276*l.* 10*s.*

9. What is the amount of 1 farthing for 28 years, at 5 per cent.?—Ans. 1 penny, nearly.

10. What is the difference between the simple and compound interests of 10*l.* for 10 years, at 10 per cent.?—Ans. 5*l.* 18*s.* 9*d.*

EXAMPLES TO NOTE 2.

11. Find the amounts of 700*l.* for 3½ years, at 3½ per cent., payable half-yearly and quarterly.—Ans. Half-yearly, 790*l.* 7*s.* 8*d.*; quarterly, 790*l.* 15*s.* 6½*d.*

12. Find the yearly, half-yearly, and quarterly interests of 500*l.* for 4 years, at 5 per cent.—Ans. Yr. 107*l.* 15*s.* 0½*d.*; hf. yr. 109*l.* 3*s.* 10½*d.*; qr. 109*l.* 19*s.* 2½*d.*

DISCOUNTING BILLS.

Definition.—A bill is said to be discounted when it is indorsed to any person, or banking company, who pays the money, deducting the interest for the time the bill has to run; therefore, to discount a bill is to procure cash for it before it becomes due.

To find the Discount.

Rule.—If the number of days be not given, reckon from the time the bill is discounted till the day on which it becomes payable (which is 3 days beyond the term of the bill); then find the discount precisely as interest is computed in *Case* 4, Simple Interest.

Note.—When the discount is taken from the sum of the bill, the balance is, in business, called the proceeds.

Remark.—Discount is but a species of interest, in which, however, mercantile practice is at variance with scientific theory. If I hold a bill for £100 which will not be due for 31 days to come, and want ready money for it, it is plain that the person who should give me £100 in cash for the bill would be a loser of the interest of the same for 31 days, and that he is therefore entitled to deduct part of the amount in cashing the bill for me. But it is as plain that if he retain the full interest upon £100 for 31 days, (which is the mercantile practice, and agreeably to the preceding rule,) he retains too much, and gives me too little; for he charges me with interest, not only upon the principal which he advances, but also upon the interest which he keeps in his own hands. He ought *equitably* to give me such a principal which being put to interest for 31 days would just amount to £100, which is found by the following rule:—Find the amount of £100 at the proposed rate, and for the given time; then, as this amount is to the sum of the bill, so is the interest of £100 for the same time to the discount required.

EXAMPLE.

Required the proceeds of a bill of £73, dated Feb. 9th, at 3 months, and discounted on April 3d.

A bill drawn Feb. 9th, at 3 months, is due May 12th.
From April 3d till May 12th is 39 days.

£73		Bill	£73	0	0
39		Discount	0	7	$9\frac{141}{365}$
657		Proceeds	£72	12	$2\frac{224}{365}$
219					
7300)	2847				
	$7s.\ 9\frac{219}{365}d.$*				

EXAMPLES.

1. Required the proceeds of a bill of £720, dated January 5th, at 4 months, and discounted on February 27th.—Ans. $713l.\ 1s.\ 11\frac{1}{3}d.$†

2. How much should a banker deduct for advancing cash on a bill of 1200*l.* for 73 days?—Ans. 12*l.*

3. What is the discount of a bill per 500*l.* for 88 days.—Ans. $6l.\ 0s.\ 6\frac{2}{3}d.$

4. A bill for 75*l.* dated 1st Sept. payable 4 months after date, was discounted 19th Nov.: how much did the holder receive?—Ans. $74l.\ 10s.\ 6\frac{2}{3}d.$

5. A bill of 700*l.* dated 7th Aug. at 5 months, was discounted 20th Sept.: what were the proceeds?—Ans. $689l.\ 5s.\ 2\frac{2}{3}d.$

6. On May 1st, the following bills were discounted at the bank:—

	l.	*s.*	*d.*	
1 bill of	716	18	4,	due May 30.
1	854	17	1,	due June 12.
1	375	16	7,	due June 29.
1	500	0	0,	due July 3.

What was the amount of the discount, and what was the present worth?—Ans. Disc. $15l.\ 2s.\ 4\frac{401}{365}d.$, and proceeds $2432l.\ 9s.\ 7\frac{331}{365}d.$

* This in business would be reckoned 7*s.* 9*d.*

† When no rate is given, 5 per cent. is understood.

EQUATION OF PAYMENTS.

In order to find the average or mean time at which two or more sums of money payable at different times may be discharged at once, without injury to either party, men in business use the following

Rule.—Multiply each debt by the time it has to run; then divide the sum of the products by the sum of the debts; the quotient is the time nearly at which all the debts ought to be paid.

EXAMPLE.

A debt of 500*l.* was to be discharged thus:—$\frac{1}{5}$ in ready money, $\frac{1}{5}$ at 3 mo., $\frac{1}{5}$ at 4 mo., $\frac{1}{5}$ at 6 mo., and the rest at 8 mo. Find the mean time for paying the whole at once.

£			
$\frac{1}{5}$) 500			
100	ready money.		
100	× 3 mo.	=	300
100	× 4 mo.	=	400
100	× 6 mo.	=	600
100	× 8 mo.	=	800
5,00		)	21,00

$4\frac{1}{5}$ mo.—Ans.

EXAMPLES.

1. If 112*l.* be payable in 84 days, 146*l.* in 76 days, 98*l.* in 38 days, and 100*l.* in 50 days; required the equated time of the whole.—Ans. $64\frac{11}{114}$ da.

2. A is indebted to B the sum of 750*l.*, which was to be paid thus: 250*l.* at the end of $1\frac{1}{2}$ years, 100*l.* at the end of 2 years, and 400*l.* at the end of 4 years: at

what time ought the whole to be discharged in one payment?*—Ans. 2 yrs. 10 mo. 24 da.

3. Find the mean time of paying the following debts: 23*l.* due in a week, 57*l.* in 3 weeks, 10*l.* in 7 weeks, and 139*l.* in 11 weeks.—Ans. $7\frac{19}{229}$ weeks.

4. Required the mean time of paying 199*l.* whereof the half becomes due in 2 months, the fifth of the remainder in 3 months after; and the rest at a year from that date.—Ans. 8 mo. 9 da.

5. One-third of a debt is to be discharged in 1 month; $\frac{1}{6}$ at three months, $\frac{1}{8}$ at 4 months, $\frac{1}{12}$ at 8 months, and the remainder at 12 months: at what time ought the whole to be paid without loss to either party?—Ans. 9 mo. 5 da.

6. Sold to A B goods payable as follows: 70*l.* on 1st Jan., 110*l.* on 2d March, 80*l.* on 5th May, 120*l.* on 20th July, 48*l.* on 27th Sept., and 52*l.* on 7th Oct.: required the mean time for paying the whole at once.—Ans. 23d May.

PROFIT AND LOSS.

Definition.—Profit and Loss is that rule which determines the gain or loss in the purchase or sale of goods.

Case 1.—When the gain or loss on one article is given, the gain or loss on a given quantity is found by multiplying by that quantity, or it may be found by the rules in Practice. And, on the contrary, when the gain or loss on any quantity is given, the gain or loss on one article is found by dividing by the quantity. Or, if the whole gain or loss, and that on one article, are given, the quantity is found by dividing by the gain or loss on one article.

* Allowing 30 days to each month, and 12 months to a year.

EXAMPLE I.

Bought 765 yards of cloth, at 15*s.* and sold it afterwards at 16*s.* 8*d.*: what did I gain?

16*s.* 8*d.* — 15*s.* = 1*s.* 8*d.* the gain per yard

Then 1*s.* 8*d.* = $\frac{1}{12}$) 765

Ans. 63*l.* 15*s.* whole gain.

EXAMPLE II.

Cleared 63*l.* 15*s.* by the sale of cloth, on which I had 1*s.* 8*d.* per yard: what quantity did I sell?

s.	*d.*		*£*	*s.*
1	8	|	63	15
3			20	
5			1275	
			3	
		5)	3825	
			765 yds.—Ans.	

Case 2.—Given the prime cost* and the profit or loss upon it, to find the profit or loss per cent.

Rule.—The prime cost : the profit or loss on it :: 100 : the profit or loss per cent.

EXAMPLE.

Bought cloth at 15*s.* per yard, and sold it at 1*s.* 8*d.* profit per yard: what was the gain per cent.?

s.		*s.*	*d.*		*£*
15	:	1	8	::	100
3		3			
45		5			$11\frac{1}{9}$ per cent.—Ans.
9		1			

* *Prime cost* is the first cost, or wholesale price of goods.

Case 3.—Given the rate per cent. and prime cost, to find the selling price.

Rule.—100 : 100 increased by the rate per cent. in case of gain, but lessened in case of loss :: the prime cost : the selling price.

EXAMPLE.

Gained $11\frac{1}{9}$ per cent. by cloth which I bought at 15*s.*: what did I sell it at?

100	:	$111\frac{1}{9}$	::	*s.* 15
9		9		10
9,00		10,00		9) 150

16*s.* 8*d.*—Ans.

Case 4.—Given the rate per cent. and selling price, to find the prime cost.

Rule.—100 increased or decreased by the rate per cent. : 100 :: the selling price : the prime cost.

EXAMPLE.

If I gain $11\frac{1}{9}$ per cent. on cloth I sold at 16*s.* 8*d.* what was the prime cost?

$111\frac{1}{9}$	:	100	::	*s.* 16	*d.* 8
9		9			9
10,00		9,00		)·150	0

15*s.*—Ans.

Case 5.—Given, two selling pieces, and the rate per cent. in proportion to one of them, to find the rate per cent. corresponding to the other.

Rule.—The price whose rate is given : 100 increased or diminished :: the other given price : a fourth number, from which subtract 100 in case of gain, but which subtract from 100 in case of loss; the remainder will be the required rate.

EXAMPLE.

By selling goods at 16*s.* 8*d.* I gained 11⅓ per cent.: what did I gain per cent. when I sold them at 17*s.* 6*d.*?

s.	*d.*				*s.*	*d.*
16	8	:	111⅓	::	17	6
12			21		12	
20,0			2331		21,0	
			2⅓			

2,0) 233,3⅓

$116\frac{13\frac{1}{3}}{20} = 116\frac{2}{3}$. (See *Case* 10, Arith. Frac.)

Then $116\frac{2}{3} - 100 = 16\frac{2}{3}$ per cent.—Ans.

Case. 6.—Given, the whole gain or loss, and the rate per cent. to find what the whole was bought and sold at.

Rule.—The rate : 100 :: the gain or loss : the buying price, and the selling price is found by adding the gain or subtracting the loss.

EXAMPLE.

Sold goods at 11⅓ per cent. profit and gained 63*l.* 15*s.*: what did I pay for them?

					£	*s.*	
11⅓	:	100	::	63	15		
9		9			9		
1,00		9,00		£573	15		buying price.—Ans.

EXAMPLES TO CASE 1.

1. Bought 428 yards of cloth, at 14*s.* 8*d.*, and afterwards sold it at 16*s.* 3*d.*: what did I gain?—Ans. 33*l.* 17*s.* 8*d.*

2. Bought 236 feet of wood, at 3*s.* 10*d.* and sold it at 3*s.* 5*d.*: what did I lose in it?—Ans. 4*l.* 18*s.* 4*d.*

3. By selling sugar at $8\frac{1}{2}d.$ per lb. which had been bought at 4 guineas per cwt. I lost 85*l.*: what quantity did I sell?—Ans. 242 cwt. 3 qrs. 12 lb.

EXAMPLES TO CASE 2.

4. Bought cloth at 3*s.* 8*d.*, and sold it at 4*d.* profit per yard: what was the gain per cent.?—Ans. $9\frac{1}{11}$ per cent.

5. Sold cloth at 15*s.* per yard value, at 1*s.* 6*d.* per yard loss: required the loss per cent.—Ans. 10 per cent.

6. How much per cent. is $2\frac{1}{2}d.$ per shilling?—Ans. $20\frac{5}{6}$ per cent.

EXAMPLES TO CASE 3.

7. Gained $9\frac{1}{11}$ per cent. by cloth, which I bought at 3*s.* 8*d.*: what did I sell it at?—Ans. 4*s.* per yard.

8. Lost 10 per cent. by cloth, which I bought at 15*s.*: what did I sell it at?—Ans. 13*s.* 6*d.* per yard.

9. Bought coffee at 4*s.* per lb.: at what must I sell it to gain $20\frac{5}{6}$ per cent.?—Ans. 4*s.* 10*d.* per lb.

EXAMPLES TO CASE 4.

10. If I gain $9\frac{1}{11}$ per cent. on cloth, which I sold at 4*s.*: what was the prime cost?—Ans. 3*s.* 8*d.*

11. Lost 10 per cent. on cloth which I sold at 13*s.* 6*d.*: what was the prime cost?—Ans. 15*s.*

12. Sold cloth at 4*s.* 10*d.* per yard, by which I cleared $20\frac{5}{6}$ per cent.: what did I buy it for?—Ans. 4*s.*

EXAMPLES TO CASE 5.

13. By selling cloth at 5*s.* I gained 12 per cent.: what did I gain per cent. by selling at 6*s.*?—Ans. $34\frac{2}{3}$ per cent.

14. By selling goods at 8*s.* I lost 14 per cent.: what did I lose by selling at 7*s.* 6*d.*?—Ans. $19\frac{3}{4}$ per cent.

15. By selling coffee at 5*s.* 3*d.* I gained 16 per cent.;

the same coffee sold afterwards at 4*s.* 6*d.*: what was lost or gained by this last price?—Ans. $\frac{4}{7}$ per cent. loss.

EXAMPLES TO CASE 6.

16. By selling goods at 5 per cent. profit, I gained 44*l.* 16*s.*: what did I pay for them?—Ans. 896*l.*

17. I lost 34*l.* 18*s.* on a quantity of cloth. at 4 per cent.: what was it bought and sold for?—Ans. Bought for 872*l.* 10*s.*, and sold for 837*l.* 12*s.*

18. By selling 386 gallons of rum at 3 per cent. loss, I lost altogether 19*l.* 18*s.* 6*d.*: what was the gallon bought and sold at?—Ans. Bought at, per gall. 1*l.* 15*s.* $5\frac{75}{193}d.$; sold at, per gall. 1*l.* 14*s.* 5*d.*

EXCHANGE.

Definition.—Exchange is the act of giving or receiving money of one country for its equivalent in that of another.

Note 1.—When the money of one nation is compared with that of another, its value, which is, in point of *intrinsic worth*, exactly equal to a given sum of the currency of the other, is termed the *par*, *rate*, or *course* of exchange.

2. *Banco*, or *bank-money*, on account of its fineness, and having no defect in weight, is superior to the *currency*, or *current money*, and their difference is called *agio*.

Remark.—The exercises in exchanges, and the arbitration of exchanges, may all be performed by Multiplication, Division, Practice, or Proportion.

Case 1.—THE NETHERLANDS.

In Holland, accounts were formerly kept in *guilders*, *stivers*, and *pfenings*; the prices of some commodities.

however, were stated in *Flemish money*, as was also the course of exchange on London and other commercial cities. Although these methods have not altogether been relinquished, the *decimal* division of money, weights, and measures, is in force, and accounts are generally kept in *florins* and *cents*.

100 cents = 20 stivers = 1 florin or guilder.
2 grotes Fl. = 16 pfenings = 1 stiver.
100 grotes Fl. = 2½ guilders = 1 rix-dollar.

Flemish Money.

12 grotes, or pence, ($\frac{3}{10}$ guild.) = 1 shilling.
20 shillings (6 guild.) = 1 pound.

Remark 1.—The French franc is permitted to circulate for 47½ cents.

2. The course of exchange at Amsterdam, December 22d, 1820, was 40*s.* 6*d.* Fl. banco, per pound sterling.

Note.—To reduce *currency* to *banco*, and the contrary.

100 + agio : 100 :: currency : banco.
100 : 100 + agio :: banco : currency.

EXAMPLE.

Reduce 560 florins banco into florins currency, agio 2 per cent.

100 : 102 :: 560 : 571 Fl. 4 st.—Ans.

Case 2.—HAMBURGH.

At Hamburgh accounts are kept in *marks*, *shillings*, and *pfenings banco*; but in exchange transactions, *pounds*, *shillings*, *and grotes Flemish*, banco are also used.

The names and subdivisions of banco and currency are the same, but the former is of considerably greater value than the latter, the agio being from 20 to 25 per cent. the par of which is about 23.

Flemish.

6 pfenings	=	1 grote, or penny.
12 grotes, or pence	=	1 shilling.
20 shillings	=	1 pound.
1 mark banco	=	32 pence.
3 marks banco	=	8 shillings.

Hambro'.

12 pfenings	=	1 sol, or shilling.
16 shillings	=	1 mark.
2 marks	=	1 dollar of Exchange.
3 marks	=	1 rix-dollar, (4*s.* 6*d.*)

Remark.—The course of Exchange at Hamburgh, on December 19, 1820, was 37*s.* Flem. banco, per pound sterling.

EXAMPLE.

How much sterling money in 300 marks, 10 shillings currency, exchange 35*s.* Flem. banco per pound sterling, agio 24 per cent.?

First, $300\frac{10}{16} = 300\frac{5}{8} = 300.625$ marks.

				curr.		banc.
Then 124	:	100	::	300.625 m.	:	242.44 m.

Flem.		Flem.		*l.*
and 35*s.*	:	242.44 m.	::	1
12		32		
420*d.*		48488		
		72732		
		775.808*d.*		
42	6	129.30133		
	7			

£18.47162 = £18 9*s.* 5*d.*—Ans.

Case 3.—FRANCE.

The French formerly kept their accounts in *livres*, *sols*, and *deniers*, but now in *francs* and *centimes*.

10 centimes = 1 décime.
10 décimes = 1 franc.
80 francs = 81 livres tournais.

Remark 1.—The words *franc* and *livre* were formerly *synonymous*, but they now denote different sums; by the latter is meant the old *livre tournais*, and by the former, the silver piece weighing 5 grammes of the new French weight established by the renowned Bonaparte.

2.—The course of exchange at Paris, December 20th, 1820, was 25 francs, 50 cents. per pound sterling.

EXAMPLE.

How much sterling money in 6580 francs, 15 centimes, exchange 25 francs, 25 centimes, per pound sterling?

fr. fr. £
25.25) 6580.15 (260.6 = £260 12*s*.—Ans.
15301
15150

Case 4.—SPAIN.

At Madrid, Malaga, Grenada, Corunna, &c. accounts are kept in *piastres*, *reals*, and *maravedis*.

Exchanges between England and Spain are, for the most part, negociated by the piastre or dollar of exchange, for which London gives a various number of pence sterling.

34 maravedis = 1 real.
8 reals = 1 piastre.
4 piastres = 1 pistole of Exchange.
375 maravedis = 1 ducat of Exchange.

Remark.—The course of exchange at Madrid on December 9th, 1820, was 30 pence sterling for 1 péso or piastre.

EXAMPLE.

In 176 ducats of Exchange, how many pounds stirling, exchange at 30 pence per piastre?

176 duc.
375

880
1232
528

34) 66000 mar.

8) 1911.765 rea.

2*s.* 6*d.* = ¼) 238.9706 pia.

£29.8713 = £29 17*s.* 5¼*d.*—Ans.

Case 5.—PORTUGAL.

In Portugal accounts are kept in *milreis* and *reis*.

1000 reis = 1 milrei.
400 reis = 1 crusado of Exchange.
480 reis = 1 new crusado.

Remark.—The course of exchange at Lisbon on December 26th, 1820, was, 50 pence sterling per melrei.

EXAMPLE.

Reduce 7649 milreis, 688 reis, into sterling money, exchange 50 pence per milrei.

40*d.* = ⅛) 7649.688

10*d.* = ¼) 1274.948
318.737

£1593.685 = £1593 13*s.* 8½*d.*—Ans.

Case 6.—LEGHORN.

At Leghorn accounts are kept in *piastres*, *soldi*, and *denari*.

12 denari = 1 soldo.
20 soldi = 1 piastre, pezza, or dollar.

Remark.—The course of exchange at Leghorn, on

September 11th, 1819, was 49¼ pence sterling for 1 pezza.

EXAMPLE.

Reduce £500 sterling into pezze, exchange at 50 pence per pezza.

£500
240

5,0*d.*) 12000,0*d.*

2400 pez.—Ans.

Case 7.—RUSSIA.

In the Russian empire accounts are kept in *rubles* and *copecs*: but Petersburgh more frequently exchanges with London by way of Hamburgh or Amsterdam, at the rate of from 40 to 50 stivers per ruble, though sometimes *directly*, and in which case the ruble varies from 4*s.* to 5*s.*

50 copecs = 1 poltin.
2 poltins = 1 ruble.
2 rubles = 1 ducat.

Remark.—The course of exchange at Petersburgh, December 1st, 1820, was 9¾ pence sterling, per ruble bank notes.

EXAMPLE.

In 1760 ducats how many pounds sterling, exchange at 48 pence per ruble?

1760 duc.
2

48*d.* = ⅕) 3520 ru.

£704.—Ans.

Case 8.—PRUSSIA and POLAND.

Accounts are kept in Polish *florins*, *groschens* and *pfenings*

18 pfenings = 1 groschen.
30 groschens = 1 florin or Polish guilder.
3 florins = 1 rix-dollar.
8 florins = 1 ducat.
5 rix-dollars = 1 Frederic d'Or.

Remark.—The course of exchange at Berlin, December 16th, 1820, was 7 rix-dol. $\frac{1}{2}$ groschen currency per pound sterling; and at Koningsberg, March 4th, 1820, 20$\frac{3}{5}$ flor. per pound sterling.

EXAMPLE.

In 749 rix-dollars, how many pounds sterling, exchange at 7 rix-dol. per pound?

7) 749 rix-dol.

£107.—Ans.

Case 9.—IRELAND.

In Ireland accounts are kept in *pounds*, *shillings*, and *pence*, which currency at *par* is to English currency as 13 to 12. Hence any sum English is converted to Irish money by adding $\frac{1}{12}$ of itself, and any sum Irish is reduced to sterling money by subtracting $\frac{1}{13}$. The exchange between England and Ireland is from 5 to 12 per cent. according to the balance of trade.

EXAMPLE I.

Reduce £264 sterling to Irish currency.

£
$\frac{1}{12}$) 264
22

£286.—Ans.

EXAMPLE II.

Reduce £286 Irish to English currency.

£
$\frac{1}{13}$) 286
22

£264.—Ans.

Case 10.—AMERICA and the WEST INDIES.

Accounts are kept in the United States either in *pounds*, *shillings*, and *pence*, currency; or, more generally, in *dollars* and *cents*.

In Spanish America, money is reckoned in *dollars*, *reals*, and *quartos*.

In the Brazils, the decimal system of Portugal is in use: and in the British West Indies and Canada, pounds, shillings, and pence, current. At Jamaica, the value of £1 currency is to that of £1 sterling as $\frac{5}{7}$ to 1; that is, £7 currency are equal to £5 sterling. In other parts of the West Indies, sterling money is worth double the currency.

Table of the Money of the United States.

10 mills = 1 cent.
10 cents = 1 dime.
10 dimes = 1 dollar, (4*s.* 6*d.* ster.)
10 dollars = 1 eagle.

EXAMPLE I.

Reduce £125 Jamaica currency to sterling.

£
125
5

7) 625

£89 5 8$\frac{4}{7}$.—Ans.

EXAMPLE II.

Reduce 2001 dollars, 50 cents, to sterling money exchange at *par*.

doll.
4*s.* = $\frac{1}{5}$) 2001.5

6*d.* = $\frac{1}{8}$) 400.3

50.0375

£450.3375 = £450 6*s.* 9*d.*—Ans.

EXAMPLES TO CASE 1.

1. Reduce 571 fl. 4 st. currency into banco agio 2 per cent.—Ans. 560 florins.

2. Reduce £500 sterling into florins, exchange 12 fl. 8 st. per pound sterling.—Ans. 6200 florins.

3. Reduce 5647 fl. 15 st. into sterling money, exchange 12 fl. 12 st. per pound sterling.—Ans. £448 4*s*. 8*d*.

EXAMPLES TO CASE 2.

4. Reduce £900 sterling to marks and shillings banco, exchange at 35½*s*. Flem. per pound sterling.—Ans. 11896*m*. 14*s*.

5. How much sterling is there in 8732 marks, 10 shillings, banco, exchange 33*s*. 10*d*. Flem. per pound sterling?—Ans. £688 5*s*. 8½*d*.

6. How many marks current in £482, exchange 33*s*. 4*d*., agio 15?—Ans. 6928*m*. 12*s*.

EXAMPLES TO CASE 3.

7. Reduce £728 15*s*. to francs and centimes, exchange at 23½ francs per pound sterling.—Ans. 17125 fr. 62½ cents.

8. What sterling money is there in 5745 francs, 75 cents, when the exchange is at 25½ fr. per pound sterling?—Ans. £225 6*s*. 5½*d*.

9. Find an equivalent in francs and centimes for 300 guineas, according to the course of exchange in 1820.—Ans. 8032 francs, 50 centimes.

EXAMPLES TO CASE 4.

10. Reduce 1090 piastres, 7 reals, to sterling money, exchange at 30 pence per piastre?—Ans. £136 7*s*. 2½*d*.

11. How many pistoles of exchange are there in £426, exchange at 41⅞ pence per piastre?—Ans. 610 pis. 1 pia. 4 rea.

12. In 610 pistoles, 1 piastre, 4 reals, how many pounds sterling, exchange at 41⅞ pence per piastre?—Ans. £426 nearly.

EXAMPLES TO CASE 5.

13. Reduce 228 milreis, 750 reis, into sterling money, exchange 50 pence per milrei. — Ans. £47 13*s*. 1½*d*.

14. Reduce £320 10*s*. 6*d*. into Portuguese money, exchange 53 pence per milrei.—Ans. 1451 mil. 433$\frac{1}{3}$ r.

15. What sum in British money is equivalent to 2926 milreis, 500 reis, exchange at 60 pence per milrei?—Ans. £731 12*s*. 6*d*.

EXAMPLES TO CASE 6.

16. Reduce £262 15*s*. to Leghorn money, exchange at 50 pence per piastre.—Ans. 1261 pias. 4 sol.

17. Reduce 1261 pezze, 4 soldi, to British money, exchange at 50 pence per dollar.—Ans. £262 15*s*.

18. In 1500 dollars, how many pounds sterling, exchange at 48 pence per piastre?—Ans. £300.

EXAMPLES TO CASE 7.

19. In 1675 rubles, 25 copecs, how much sterling, exchange at 50 stivers* per ruble, and 33*s*. 4*d*. Flem. per pound sterling?—Ans. £418 16*s*. 3*d*.

20. In £1349 15*s*. how many rubles, rating the ruble at 49 stivers, and exchange at 33*s*. 9½*d*. Flem. per pound sterling?—Ans. 5584 ru. 93 cop.

EXAMPLES TO CASE 8.

21. In 500 rix-dollars, how many pounds sterling, allowing 7 rix-dollars, and 1 groschen per pound. —Ans. £71 6*s*. 3½*d*.

22. In £71 6*s*. 4*d*. how many rix-dollars, allowing 7 rix-dollars and 1 groschen to the pound sterling?—Ans. 500$\frac{11}{20}$ rix-d.

23. In 8796 florins, exchange at 280 groschen per pound Flem. and 35*s*. 8*d*. Flem. per pound sterling, how many pounds sterling?—Ans. £528 9*s*. 3½*d*.

* A stiver = 2 pence Flem.

EXAMPLES TO CASE 9.

24. In £3847 16*s.* 8*d.* British, how much Irish currency, exchange at 10¾*d.* per cent.? — Ans. £4261 9*s.* $6\frac{1}{10}$*d.*

25. Dublin draws upon London for £138 5*s.* 3*d.* Irish currency, exchange at 11⅛ per cent. for how much sterling must Ireland be credited? — Ans. £392 12*s.* 5*d.*

EXAMPLES TO CASE 10.

26. Reduce £375 5*s.* sterling to currency, exchange at 160 per cent.?—Ans. £562 17*s.* 6*d.*

27. Required an equivalent in American dollars for £215 8*s.*, exchange at 4*s.* 7½*d.* per dollar.—Ans. 931 doll. 46 cents, nearly.

28. If 500 American eagles be paid at New York for a draft on London, what sum sterling will pay it, exchange at 4*s.* 9*d.* per dollar.—Ans. £1187 10*s.*

ARBITRATIONS OF EXCHANGE.

If the courses of exchange were always equivalent to each other, there would be no advantage in transmitting money through different countries; the *direct* exchange would be preferred, it being attended with less expense as well as less delay. But the courses are always fluctuating, and are often in such a situation that money may be drawn or remitted to considerable advantage by passing through several places. It is manifest, from a little consideration, that the less value the money of a merchant's own country bears abroad, the more he will gain by drawing, and lose by remitting; and the greater its value the contrary: whence, *when the exchange is made upon British money*, Britain will gain by remitting when the course is high, and lose by drawing; but when the course is low,

Britain will gain by drawing, and lose by remitting. But, *if the exchange be transacted by the foreign price*, the contrary. Therefore, *to gain* when the exchange is by *British money*, remit when the course is high, and draw when it is low: when upon the *foreign price*, draw when the course is high, and remit when it is low.

Arbitration may be either Simple or Compound.

Case 1.—SIMPLE ARBITRATION.

Definition.—Simple Arbitration is the method of finding such a rate of exchange between any two places as shall be in proportion with the rates assigned by each of them in a third place.

Rule.—Proceed by Proportion, having the first and second terms of the same kind, and the third term the same as the arbitrated *par* required.

EXAMPLE.

Suppose the course of exchange from London to Madrid be 33*d.* per piastre, and from London to Lisbon be 67*d.* per milrei; required the arbitrated par between Madrid and Lisbon, or, which is the same thing, how many reis should be given for 1 piastre?

d.		*d.*		reis.
67	:	33	::	1000
				33

67) 33000 (492$\frac{4}{67}$ reis.—Ans.
268

622
603

190
134

56

Case 2.—COMPOUND ARBITRATION.

Definition.—Compound Arbitration is the me-

thod of finding the most eligible medium of negociation in cases where more than three places are concerned.

Rule.—Arrange the several courses of exchange into antecedents and consequents, as directed in Proportion, and place all the antecedents in a left and the consequents in a right-hand column, in such a manner that the first consequent may be of the same denomination as the second antecedent, and the second consequent as the third antecedent, and so on throughout the whole; and, at the same time, let the first antecedent be of the same kind with the last consequent.

2. When the odd number falls on the right-hand, place all the consequents, with the sign of multiplication between them, above a horizontal line, and all the antecedents in the same manner below it; but when the odd number falls on the left-hand, the contrary; and abbreviate as recommended in Proportion, which will give the value of the sum required.

3. Calculate the value of the sum by the direct exchange, or by any circular exchange, and a comparison of these values will determine the most advantageous medium of negociation.

EXAMPLE.

If London remit £800 to Cadiz by way of Amsterdam, at 33*s.* 4*d.* Flem. per pound sterling, to be forwarded to Paris, at 52*d.* Flem. per crown; thence to Vienna at 100 crowns for 60 ducats; thence to Cadiz, at 370 maravedies per ducat; how many piastres will the £800 amount to in Spain, and whether will the circular or direct exchange be most advantageous, the exchange between London and Cadiz being 36*d.* per dollar of exchange?

	ante.		conse.
First,	£1	=	400*d.*
	52*d.*	=	1 cr.
	100 cr.	=	60 duc.
	1 duc.	=	370 mar.
	375 mar.	=	1 piastre.
			£800.

Then, $\frac{\overline{4}\ \ \overline{4}\ \ \overline{32}}{\overline{400} \times \overline{60} \times 370 \times \overline{800}} = \frac{4 \times 370 \times 32}{13}$

$\frac{\overline{52}}{13} \times \frac{\overline{100}}{} \times \frac{\overline{375}}{15}$

pia. mar.

$= 3643.28\frac{4}{13}$ by the circular exchange.

£800
20

36*d*. = 3*s*.) 16000

5333 pia. 125 mar. by the direct exchange.
3643 pia. $28\frac{4}{13}$ mar. by the circular exchange.
1690 pia. $96\frac{2}{13}$ mar. in favour of the direct exchange.

EXAMPLES TO CASE I.

1. If the exchange between London and Amsterdam be 33*s*. 6*d*. per pound sterling, and between London and Paris 32*d*. per crown: what should be the par of exchange between Amsterdam and Paris?—Ans. 4*s*. $5\frac{3}{5}$*d*.

2. When London exchanges with Paris at £1 sterling for 24 francs, 40 cents, and with Amsterdam at 37*s*. 6*d*. Flem. per pound sterling; what should the course of exchange be between Paris and Amsterdam?—Ans. 45 fr. 75 cents.

3. Amsterdam changes on London at 34*s*. 2*d*. and on Lisbon at 50*d*. for 400 reis: how ought the exchange to go between London and Lisbon?—Ans. 6*s*. $1\frac{7}{41}$*d*. per milrei.

4. If the course of exchange between London and Madrid be 40*d*. sterling for a dollar of exchange, and between London and Oporto 60*d*. sterling for a milrei; what is the arbitrated rate of exchange between Madrid and Oporto.—Ans. $2666\frac{2}{3}$ reis per pistole.

EXAMPLES TO CASE 2.

5. If the course of exchange between London and Amsterdam be 11 florins, 14 stivers, between Amsterdam and Paris 53 groots for 3 francs, and between Paris and Madrid 15 francs for 1 pistole; what is the arbitrated rate of exchange between London and Madrid.—Ans. 1 pis. $24\frac{13}{28}$ rea. per pound sterling.

6. If Hamburgh remit to Paris 4 shillings Flem. for 3 francs; Paris remit to Madrid at 15 francs for a pistole of exchange; Madrid to Lisbon at a pistole of exchange for 2500 reis; and London draw upon Lisbon at 58*d.* sterling for a milrei; what is the corresponding exchange between Hamburgh and London? —Ans. 33*s.* $1\frac{7}{9}$*d.* Flem. per pound sterling.

7. If Paris remit to Amsterdam 9100 crowns, by way of London, at 30*d.*; thence to Rome at 65*d.* per stamped crown; thence to Venice at 100 stamped crowns per 140 ducats; thence to Leghorn at 100 ducats per 100 pias; and thence to Amsterdam at 88*d.* per pias; how many guilders will be received at Amsterdam; and how many crowns will be remitted to Paris, exchange 54*d.*? — Ans. 12936 gs. and $9582\frac{2}{3}$ cr.

INVOLUTION.

Definition.—When any given number is multiplied by itself to any assigned number of products, the process is called Involution.

Note 1.—The number to be involved is called the *root*, or first power.

2. The first power multiplied by itself gives the second, or square.

3. The second power multiplied by the first gives the third, or cube, &c.

4. But the second power multiplied by itself will give the fourth; the fourth power multiplied by itself will give the 8th, &c.

5. When the given root to be involved is a fraction, both members of it must be involved; and, if it be a mixed number, it must first be reduced either to an improper fraction, or the fractional part to a decimal before involving.

EXAMPLE I.

Involve 1.02 to the sixth power.

1.02 root, or 1st power.
1.02

204
102

1.0404 = the 2d power, or square.
1.02

20808
10404

1.061208 = the third power, or cube.
1.061208

8489664
12734496
6367248
1061208

1.126162419264 = the 6th power.—Ans.

EXAMPLE II.

Involve $\frac{2}{3}$ to the eighth power.

1st. 2d. 4th. 8th.
$\frac{2}{3} = \frac{4}{9} = \frac{16}{81} = \frac{256}{6561}$.—Ans.

EXAMPLES.

1. What is the cube of 32?—Ans. 32768.

2. What is the square of 1.157625? — Ans. 1.340095640625.

3. What is the fourteenth power of 1.05?—Ans. 1.979931599439397388305664062 5.

4. What is the ninth power of $\frac{3}{7}$?—Ans. $\frac{2441406625}{74711820 96}$.

5. What is the square of $67\frac{3}{4}$?—Ans. 4590.0625.

6. What is the cube of $\frac{3}{8}$ of $\frac{4}{7}$ of $13\frac{1}{4}$?—Ans. $\frac{6391228661}{251763097 6}$.

EVOLUTION.

Definition.—Evolution, or the Extraction of Roots, is the reverse of Involution, being the method of finding the root or first power of any assigned higher power.

Note 1.—A *rational root* is that which is perfectly accurate.

2. An *irrational-root* or *surd* is that which only approximates to the rational root.

1. *Square Root.*

Rule 1.—Prepare the given power by separating it into periods of two figures each from the right towards the left in integers, and from the decimal point towards the right in decimals.

2. Subtract the greatest square that can be taken from the first period, and set its root as a quotient figure on the right, as in common Division.

3. To the remainder annex the next period for a dividend.

4. Place the double of the root already found on the left of the dividend for a divisor, by which divide the dividend, omitting the place of units, and place the result both in the root, and on the right of the divisor; then by it multiply the divisor thus completed, and subtract the product from the dividend, then to the

remainder annex the following period for a new dividend.

5. To the completed divisor add the figure last put in the root; the sum is a new divisor, with which proceed as before.

Note.—When the figures in the given power are exhausted, the operation may be continued to any length by annexing ciphers.

EXAMPLE.

What is the square root of 5314.41?

```
5314.41 ( 72.9.—Ans.
49
---
142 ) 414
      284
-------
1449 ) 13041
       13041
-------
```

2. *Cube Root.*

As the Rules generally given for extracting the cube root are exceedingly difficult to be remembered, except by those who are frequently in the habit of extracting, the following humble doggerel, it is presumed, will not only be got by heart sooner, but will remain longer on the memory of the pupil.

First, prepare the given power by separating it into periods of three figures each, from the right towards the left in integers, and from the decimal point towards the right in decimals. Then,

Find nearest cube to period first,
  Its root to quotient send;
Subtract this cube, and then annex
  Next part for dividend.
Three hundred times the square of root
  Trial divisor make,
By which divide the dividend
  And quotient to root take;
Then former part of root express
  By this, and then by thirty,

Which under first divisor place
As second of its party.
The last root's square place under these;
The sum of all then take,
Which sum, if right you have produced,
Will *full divisor* make;
You multiply divisor now
By digit at root's end;
Subtract the product, and next part
Annex for dividend;
With this, as in the second case,
And so with all the rest:
If right you do proceed, you'll find
The proper root express'd.

Note 1.—When the dividend is, at any stage of the extraction, too small to admit of division by the divisor, annex a cypher in the root, bring down the next period for a new dividend, annex two additional cyphers to the first part of the divisor, and one to the second; then proceed as before.

2. The cube root of a fraction is found by taking the root of both its terms, if it can be done exactly; if not, reduce the fraction to a decimal, and extract the root, as before.

EXAMPLE.

What is the cube root of 705919.947264?

705919.947264 (89.04—Ans.
512

tr. div.
$8^2 \times 300 =$ 19200) 193919
$8 \times 9 \times 30 =$ 2160
$9^2 =$ 81

comp. div. $21441 \times 9 =$ 192969

$89^2 \times 300 = 237630000$*) 950947264
$89 \times 4 \times 30 =$ 106800
$4^2 =$ 16

$237736816 \times 4 = 950947264.$

* See *Note* 1.

EXAMPLES TO CASE 1.

1. What is the square root of 576?—Ans. 24.
2. What is the square root of 788544?—Ans. 888.
3. What is the square root of $\frac{81}{121}$?—Ans. $\frac{9}{11}$.
4. What is the square root of $\frac{3}{4}$?—Ans. .8660254.
5. The area of a circle is .7854; required the side of a square of the same area?—Ans. .886228.
6. A clergyman's glebe consists of four fields; the first, 2 acres, 3 roods, 4 poles; the second, 3 ac. 1 ro. 20 po.; the third, 1 ac. 15 po.; and the fourth, 4 ac. 3 ro. 24 po.: he wants a square field in exchange equal in area to all the four: required the length of its side?—Ans. 44.07947 poles.

EXAMPLES TO CASE 2.

7. Find the cube root of 1728.—Ans. 12.
8. Find the cube root of 387420.489.—Ans. 72.9.
9. Find the cube root of $\frac{64}{125}$.—Ans. $\frac{4}{5}$.
10. Find the cube root of $\frac{3}{8}$.—Ans. .816.
11. A stone of the form of a cube contains 21952 solid feet: find the area of one of its sides.—Ans. 784 so. feet.
12. The solidity of a sphere is 11390.625 inches; required the lineal side of a cube of equal solidity?—Ans. 22.5 in.

POSITION.

Definition.—Position is the method by which from a supposed number or numbers the true one is found.

1.—Single Position.

Rule.—Take any number at pleasure, and if it answer the conditions of the question, it is the number

sought; if not, as the result of the supposition is to the given result, so is the number supposed to the answer.

EXAMPLE.

A person, after spending $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{6}$, of a legacy finds he has £100 left; what was the amount of the legacy?

Sup. 240

$\frac{1}{3} = 80$
$\frac{1}{4} = 60$
$\frac{1}{6} = 40$

180

60 : 100 :: 240
4 4

£400.—Ans.

Fractionally.

$\frac{1}{3} + \frac{1}{4} + \frac{1}{6} = \frac{3}{4}$ spent.
$\therefore$ £100 = the remaining $\frac{1}{4}$.
And £400 = the legacy.

2.—DOUBLE POSITION.

Rule 1.—Take any two numbers at pleasure, and perform the same operations with each of them as the question directs; if neither of them produce a result agreeing with that in the question, mark the error of each result with the sign + or —, according as it is an error of *excess* or *defect*.

2.—Then multiply the *first supposition* by the *second error*, and the *second supposition* by the *first error*; and when the signs are both +, or both —, divide the *difference of the products* by the *difference of the errors*; but, when one of the signs is + and the other —, divide the *sum of the products* by the *sum of the errors*, for the answer.

EXAMPLE.

Three men bought a ship for £760, for which B paid £10 more than A, and C as much as both together: how much did they pay individually?

1. Sup. A paid	200	2. Sup. A paid	120
Then B	210	Then B	130
And C	410	And C	250
	820		500
	760		760
1st error	60 +	2d error	260 −

Then $200 \times 260 = 52000$
And $120 \times 60 = 7200$

$320 \;)\; 59200$

185 paid by A.
195 ... by B.
380 .... by C.

760—Proof.

EXAMPLES TO CASE 1.

1. A, B, and C, were disputing about their ages; said A, I am certainly older than B by $\frac{1}{8}$ of C's age. This I know, said B, that my age is just $\frac{3}{4}$ of C's. And I also know, said C, that our ages together is just 130. Required the particulars?—Ans. A $54\frac{1}{7}$ yrs.; B $32\frac{1}{2}$ and C $43\frac{1}{3}$.

2. A person finds, after paying away $\frac{1}{3}$ and $\frac{1}{4}$ of the produce of a lottery ticket, he has £275 left, what was the amount of his prize?—Ans. £660.

3. How many trees are in an orchard, $\frac{1}{2}$ of which bear apples, $\frac{1}{5}$ pears, and $\frac{1}{8}$ plums, and the rest, which are 30-cherries?—Ans. 240 trees.

4. Find a number to which if you add $\frac{7}{9}$ of itself, the result will be 36.—Ans. $33\frac{3}{4}\frac{3}{5}$.

5. It is required to divide 108 into three such parts that $\frac{1}{2}$ of the first, $\frac{1}{3}$ of the second, and $\frac{1}{4}$ of the third,

shall all be equal to each other?—Ans. 24, 36, and 48.

6. Two travellers set out at the same time from London and York, a distance of 150 miles; the one travels 8 miles per hour, and the other 7: how many hours must elapse before they meet, and how many miles will each have travelled?—Ans. 10 hours; and the one will travel 80 miles, and the other 70.

EXAMPLES TO CASE 2.

7. The sum of £24 was paid with guineas and half-crowns, and the number of pieces paid was 44: what was the number of each kind?—Ans. 20 gs. and 24 hf.-cr.

8. What number is that which being doubled, and 16 added to the product, the sum will be 56?—Ans. 20

9. A, B, and C, playing at roulette, in St. James's, the money staked was 196 sovereigns; but, being surprised by the officers of the police, each seized as many as he could: A got a certain quantity unknown; B as many as A, and 16 more; and C the sixth part of both their sums: how many had each?—Ans. A had 76; B 92; and C 28.

10. Two persons, C and D, play at Hazard; C stakes 8*s.* and D 6*s.* every game; if, after playing 28 games, they find that they have neither won nor lost, what number of games did each win?—Ans. C won 12 and D 16.

11. Divide 90 into four such parts, that the first increased by 2, the second diminished by 2, the third multiplied by 2, and the fourth divided by 2, may all be equal.—Ans. 18, 22, 10, and 40.

12. Divide 100 into four such parts, that the first increased by 3, the second diminished by 4, the third multiplied by 5, and the fourth divided by 6, may each be equal to one another.—Ans. $9\frac{3}{41}$, $16\frac{3}{41}$, $21\frac{7}{41}$, and $72\frac{4}{41}$.

J. and C. Adlard, Printers, 23, Bartholomew Close.